20710-214	TUTTOETS 9 / *TUT TEST 9*	2023
Voorl's en Van / *Init's and Surname:*	MEMO	
Studentenommer/ *Student number:*		

10/10

$$A = \begin{bmatrix} 5 & 0 & 5 \\ 0 & 5 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

Vind al die eiewaardes van A en vind slegs een eievektor, maar sê by watter eiewaarde daardie eievektor pas. Skryf die antwoorde in die spasies hieronder.

Find all the eigenvalues of A and find only one eigenvector. You must, however state to which eigenvalue this eigenvector belongs. Write your answers in the spaces below.

$$\left|\begin{matrix} 5-\lambda & 0 & 5 \\ 0 & 5-\lambda & 1 \\ 4 & 1 & 9-\lambda \end{matrix}\right| \begin{matrix} 5-\lambda & 0 \\ 0 & 5-\lambda \\ 4 & 1 \end{matrix}$$

$$-20(5-\lambda) - 1(5-\lambda) - 0 + (5-\lambda)(\lambda^2 - 14\lambda + 45) + 0 + 0$$

$$= (5-\lambda)(-20 - 1 + \lambda^2 - 14\lambda + 45) = (5-\lambda)(\lambda^2 - 14\lambda + 24)$$

$$= (5-\lambda)(2-\lambda)(12-\lambda) = 0 \qquad \lambda_1 = 12,\ \lambda_2 = 5,\ \lambda_3 = 2$$

$\boxed{\lambda_1 = 12}$

$$\begin{bmatrix} -7 & 0 & 5 \\ 0 & -7 & 1 \\ 4 & 1 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} -7 & 0 & 5 \\ 0 & -7 & 1 \\ 0 & 1 & -\frac{1}{7} \end{bmatrix} \longrightarrow \begin{bmatrix} -7 & 0 & 5 \\ 0 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 0 & 5 \\ 0 & -7 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_2 + \mu = 0,\ x_2 = \frac{\mu}{7}$

$-7x_1 + 5\mu = 0,\ x_1 = \frac{5\mu}{7}$

Let $\mu = 7$.

$$\underline{x}_1 = \begin{bmatrix} 5 \\ 1 \\ 7 \end{bmatrix}$$

$\lambda_1 =$	12
$\lambda_2 =$	5
$\lambda_3 =$	2

eigenvector of $\lambda = \boxed{12}$ is $\begin{bmatrix} 5 \\ 1 \\ 7 \end{bmatrix}$

ALTERNATIVE :

$\boxed{\lambda_2 = 5}$

$$\begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 1 \\ 4 & 1 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 4 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ \mu \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_3 = 0$, $x_3 = 0$, $4x_1 + \mu = 0$ $\quad x_1 = -\frac{1}{4}\mu$

Let $\mu = 4$, $\underline{x}_2 = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$

$\boxed{\lambda_3 = 2}$

$$\begin{bmatrix} 3^{4} & 0 & 5^{5\cdot 4/3} \\ 0 & 3 & 1 \\ 4 & 1 & 7 \end{bmatrix} \xrightarrow{LU} \begin{bmatrix} 3 & 0 & 5 \\ 0 & 3 & 1 \\ 0 & 1 & \frac{1}{3} \end{bmatrix}$$

$$\xrightarrow{LU} \begin{bmatrix} 3 & 0 & 5 \\ 0 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_2 + \mu = 0$

$x_2 = -\frac{1}{3}\mu$

$3x_1 + 5\mu = 0$, $x_1 = -\frac{5}{3}\mu$

Let $\mu = 3$

$$\underline{x}_3 = \begin{bmatrix} -5 \\ -1 \\ 3 \end{bmatrix}$$