20710-214	TUTTOETS 8 / TUT TEST 8	2023
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Studentenommer/ Student number:		

Skryf die oorbepaalde stelsel hieronder in matriksvorm en doen gereduseerde QR-ontbinding van die matriks. Vind dan die kleinste-kwadrate-oplossing van die stelsel.
Wenk: $R\tilde{x} = \bar{Q}^T \mathbf{b}$.

Write the over determined system below in matrix form and then do reduced QR decomposition of the matrix. Then find the least squares solution of the system.
Hint: $R\tilde{x} = \bar{Q}^T \mathbf{b}$.

$$\begin{aligned} 8x + 0.5y &= 4.5 \\ -2x + y &= 4.5 \\ 16x + 10y &= 13.5 \end{aligned}$$

Red-QR

$$A = \begin{bmatrix} 8 & \frac{1}{2} \\ -2 & 1 \\ 16 & 10 \end{bmatrix}$$

$$q_1 = \frac{1}{18}\begin{bmatrix} 8 \\ -2 \\ 16 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}$$

$$b = \frac{1}{2}\begin{bmatrix} 9 \\ 9 \\ 27 \end{bmatrix}$$

$$R = \begin{bmatrix} 18 & 9 \\ 0 & 4.5 \end{bmatrix}$$

$$b' = b - q_1(q_1^T b)$$

$$= \begin{bmatrix} 0.5 \\ 1 \\ 10 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix}\left(\frac{1}{9}[4 \ -1 \ 8]\begin{bmatrix} 0.5 \\ 1 \\ 10 \end{bmatrix}\right) = \begin{bmatrix} 0.5 \\ 1 \\ 10 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 4 \\ -1 \\ 8 \end{bmatrix} 9$$

$$= \begin{bmatrix} -3.5 \\ 2 \\ 2 \end{bmatrix} \qquad \|b'\| = 4.5 \qquad q_2 = \frac{1}{4.5}\begin{bmatrix} -3.5 \\ 2 \\ 2 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -7 \\ 4 \\ 4 \end{bmatrix}$$

LS-sol:

$$\bar{Q}^T b = \frac{1}{9}\begin{bmatrix} 4 & -1 & 8 \\ -7 & 4 & 4 \end{bmatrix}\begin{bmatrix} 4.5 \\ 4.5 \\ 13.5 \end{bmatrix} = \begin{bmatrix} 13.5 \\ 4.5 \end{bmatrix} \checkmark$$

$$\bar{Q} = \frac{1}{9}\begin{array}{|c|c|} \hline 4 & -7 \\ \hline -1 & 4 \\ \hline 8 & 4 \\ \hline \end{array} \checkmark\checkmark \qquad R = \begin{array}{|c|c|} \hline 18 & 9 \\ \hline 0 & 4.5 \\ \hline \end{array} \checkmark\checkmark \qquad \begin{array}{|c|} \hline x = 0.25 \\ \hline y = 1 \\ \hline \end{array} \checkmark$$

$$\begin{bmatrix} 18 & 9 \\ 0 & 4.5 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13.5 \\ 4.5 \end{pmatrix} \qquad \begin{aligned} y &= 1 \\ 18x + 9(1) &= 13.5, \qquad x = 0.25 \end{aligned}$$