$\frac{10}{10}$

20710-214	TUTTOETS 7 / *TUT TEST 7*	2023
Voorl's en Van / *Init's and Surname:*	MEMO	
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1 Vind die kleinste-kwadrate-oplossing van die volgende oorbepaalde stelsel deur die normaalvergelykings op te los.

Find the least squares solution of the following over determined system by solving the normal equations.

$$\begin{array}{rcrcr} -x & - & 2y & = & -4 \\ & & y & = & 1 \\ -2x & - & 5y & = & -8 \\ x & + & 2y & = & 3 \\ x & + & 2y & = & 2 \end{array}$$

$A = \begin{bmatrix} -1 & -2 \\ 0 & 1 \\ -2 & -5 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} -4 \\ 1 \\ -8 \\ 3 \\ 2 \end{bmatrix}$

$A^TA = \begin{bmatrix} -1 & 0 & -2 & 1 & 1 \\ -2 & 1 & -5 & 2 & 2 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 0 & 1 \\ -2 & -5 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 16 \\ 16 & 38 \end{bmatrix}$ ✓

$A^T\underline{b} = \begin{bmatrix} -1 & 0 & -2 & 1 & 1 \\ -2 & 1 & -5 & 2 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \\ -8 \\ 3 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 25 \\ 59 \end{bmatrix}$ ✓

$\check{\underline{x}} = (A^TA)^{-1}A^T\underline{b}$ ✓

$= \frac{1}{10} \begin{bmatrix} 38 & -16 \\ -16 & 7 \end{bmatrix} \begin{bmatrix} 25 \\ 59 \end{bmatrix}$

$= \frac{1}{10} \begin{bmatrix} 6 \\ 13 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 1.3 \end{bmatrix}$

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$x =$ 0.6 ✓
$y =$ 1.3 ✓

Daar is nog 'n probleem agterop.

There is another problem overleaf.

2 'n Stel datapunte (x_j, y_j) met $j = 1, 2, 3, ..., N$ word gegee en die volgende parabool moet daarop gepas word:

A set of data points (x_j, y_j) with $j = 1, 2, 3, ..., N$ is given and the following parabola must be fitted to it:

$$y = a + bx^2$$

Vind formules vir a en b deur die algemene kleinste-kwadrate-oplossing te bepaal. Jy mag afkortings vir die sommasies gebruik, byvoorbeeld skryf $\sum x^2$ as 'n afkorting vir $\sum_{j=1}^{N} {x_j}^2$, ens.

Find formulae for a and b by calculating the general least squares solution. You may use abbreviations for summations, for example write $\sum x^2$ as an abbreviation for $\sum_{j=1}^{N} {x_j}^2$, etc.

$y_1 = a + bx_1^2$

$y_2 = a + bx_2^2$

$\vdots$

$$\begin{bmatrix} 1 & x_1^2 \\ 1 & x_2^2 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix}.$$

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Normal eqn's: $A^TA\tilde{x} = A^T\underline{b}$

$$\begin{bmatrix} 1 & 1 & 1 & \cdots \\ x_1^2 & x_2^2 & \cdots \end{bmatrix} \begin{bmatrix} 1 & x_1^2 \\ 1 & x_2^2 \\ \vdots & \vdots \end{bmatrix} = \begin{bmatrix} \Sigma 1 & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^4 \end{bmatrix} = \begin{bmatrix} N & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^4 \end{bmatrix} \checkmark$$

$$A^T\underline{b} = \begin{bmatrix} 1 & 1 & 1 \\ x_1^2 & x_2^2 & \cdots \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma x^2 y \end{bmatrix} \checkmark$$

$$\tilde{x} = (A^TA)^{-1}A^T\underline{b} = \frac{1}{N\Sigma x^4 - (\Sigma x^2)^2} \begin{bmatrix} \Sigma x^4 & -\Sigma x^2 \\ -\Sigma x^2 & N \end{bmatrix} \begin{bmatrix} \Sigma y \\ \Sigma x^2 y \end{bmatrix}$$

$$= \frac{1}{N\Sigma x^4 - (\Sigma x^2)^2} \checkmark \begin{bmatrix} \Sigma x^4 \Sigma y - \Sigma x^2 \Sigma x^2 y \\ -\Sigma x^2 \Sigma y + N\Sigma x^2 y \end{bmatrix}$$

$$\boxed{a = \frac{\Sigma x^4 \Sigma y - \Sigma x^2 \Sigma x^2 y}{N\Sigma x^4 \; (\Sigma x^2)^2}} \checkmark$$

$$\boxed{b = \frac{N\Sigma x^2 y - \Sigma x^2 \Sigma y}{N\Sigma x^4 - (\Sigma x^2)^2}} \checkmark$$