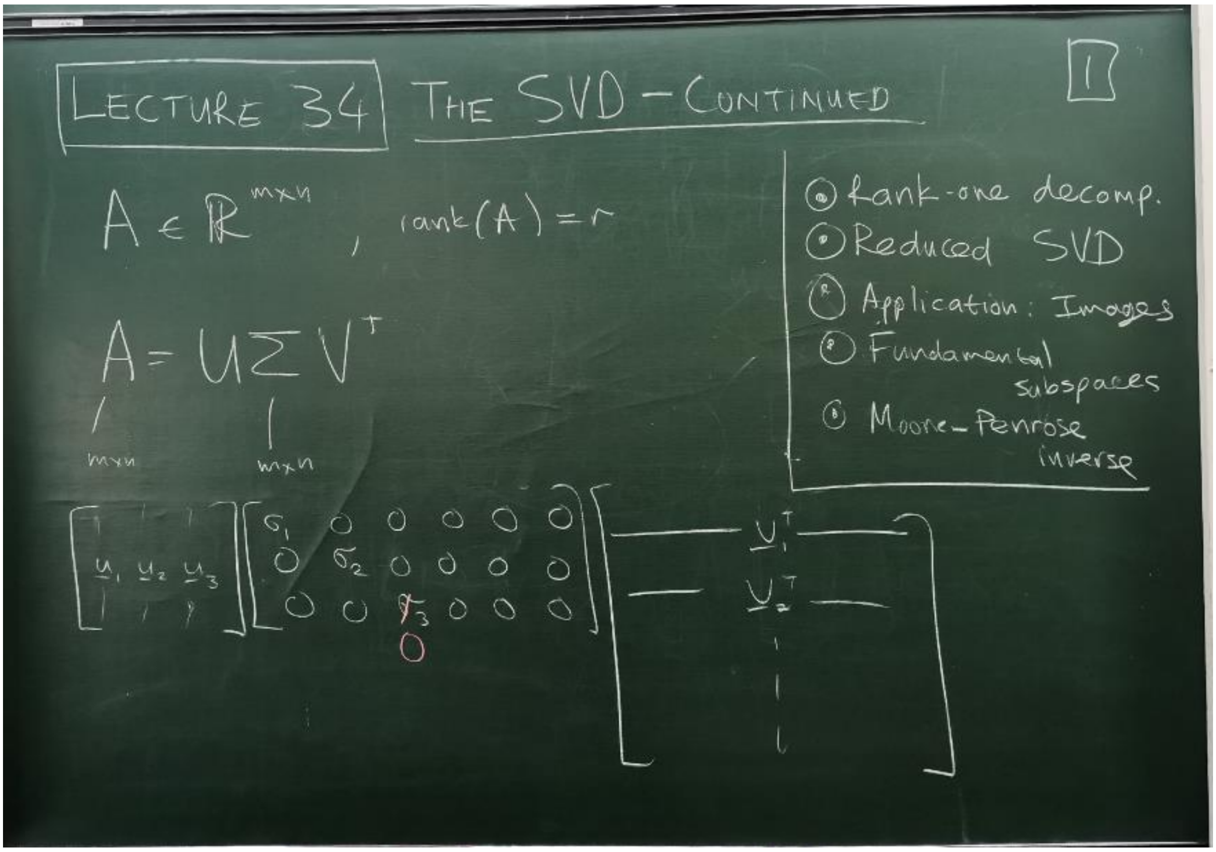
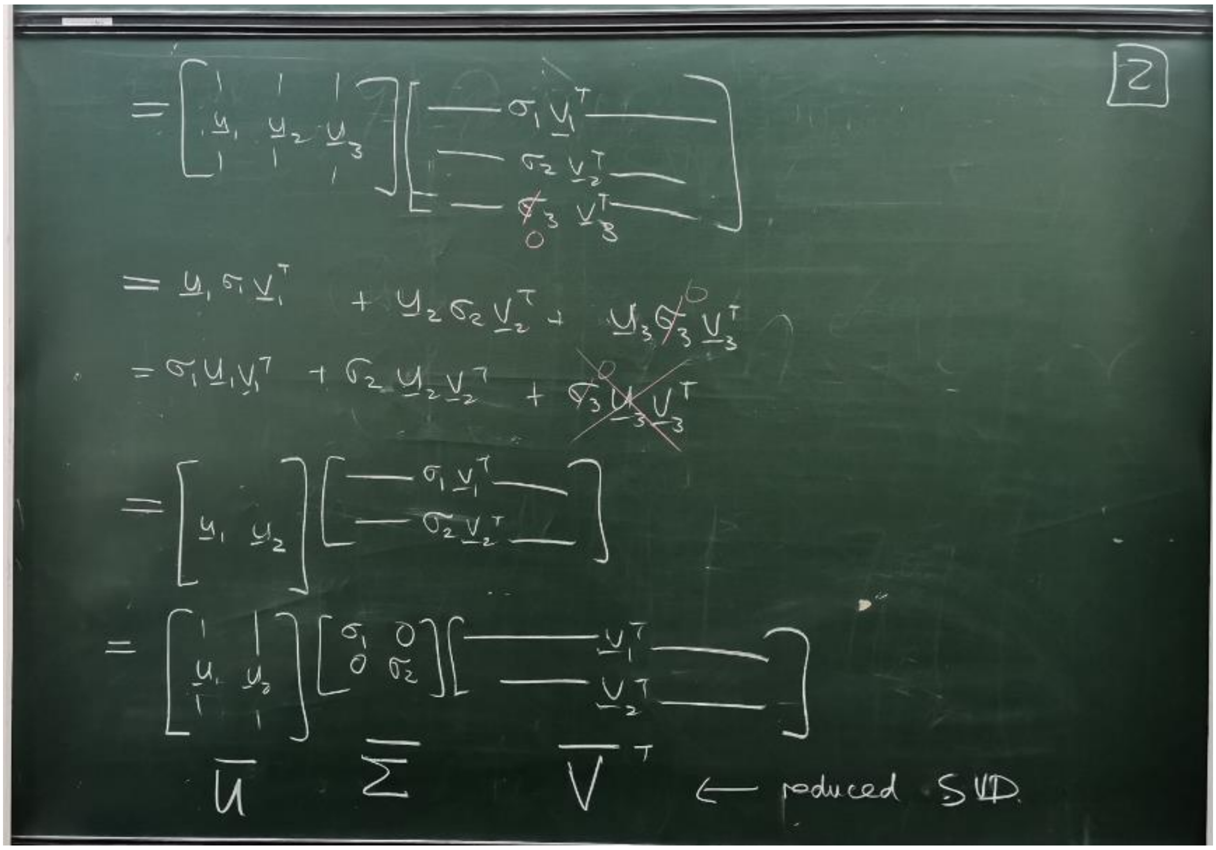
AM214-2023: LECTURE 34





$\boxed{3}$

$(3\times 2)\ (2\times 6) = (3\times 6)$

$A \in \mathbb{R}^{m\times n}$

$$= \underset{(m\times r)}{\bar{U}}\ \underset{(r\times r)}{\bar{\Sigma}}\ \underset{(r\times n)}{\bar{V}^T}$$

$\bar{U}, \bar{V}$ are reduced orthogonal matrices.

lossless / lossy compression

$$A = \begin{bmatrix} 251 & 256 & & \\ & & 10 & 12 \end{bmatrix}$$

$\boxed{4}$

$$A = 45000\, u_1 v_1^T + 6300\, u_2 v_2^T$$

How much saved (40 of the 257 singular values)

$\underbrace{260\times 40}_{\text{col's of } U}$ $\quad \underbrace{257\times 40}_{\text{col's of } V}$ $\quad 40$

5

THE FUNDAMENTAL SUBSPACES

$A \in \mathbb{R}^{m \times n}$

$\mathrm{rank}(A) = r$.

SPACE	LIES IN	DIM	ORTHOGON
Row	$\mathbb{R}^n$	r	
Null	$\mathbb{R}^n$	$n-r$	
Col	$\mathbb{R}^m$	r	
Leftnull	$\mathbb{R}^m$	$m-r$	

6

$A = U \Sigma V^T$

$= \sigma_1 \underline{u}_1 \underline{v}_1^T + \dots$

$$\begin{bmatrix} \vert & \vert & \\ \underline{a}_1 & \underline{a}_2 & \cdots \\ \vert & \vert & \end{bmatrix} = \begin{bmatrix} \vert & \vert & \\ \underline{u}_1 & \underline{u}_2 & \cdots \\ \vert & \vert & \end{bmatrix} \begin{bmatrix} \sigma_1 v_{11} & \sigma_1 v_{12} & \cdots \\ - & \sigma_2 \underline{v}_2^T & - \\ & & \end{bmatrix}$$

Col space

$\underline{a}_1 = \underline{u}_1 \sigma_1 v_{11} + \underline{u}_2 \sigma_2 v_{12} + \underline{u}_3 \sigma_3 v_{13} \dots \underline{u}_r \sigma_r v_{1r}$.

$\underline{a}_2 = \underline{u}_1 \sigma_1 v_{21} + \dots$

Each column of A is a linear combination of

$\{\underline{u}_1, \underline{u}_2 \dots \underline{u}_r\}$

First r columns of U is a basis for $\mathcal{C}(A)$. $\boxed{7}$

Left null sp.

If $y^T A = \underline{0}^T$, then $y \in \mathcal{L}(A)$

$$A = U \Sigma V^T$$

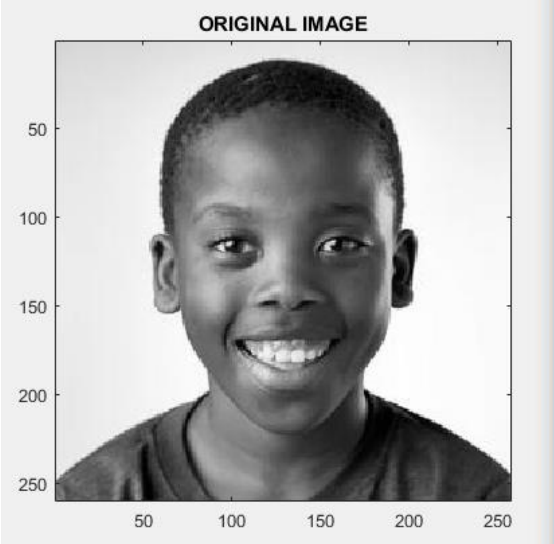
$$U^T A = \Sigma V^T$$

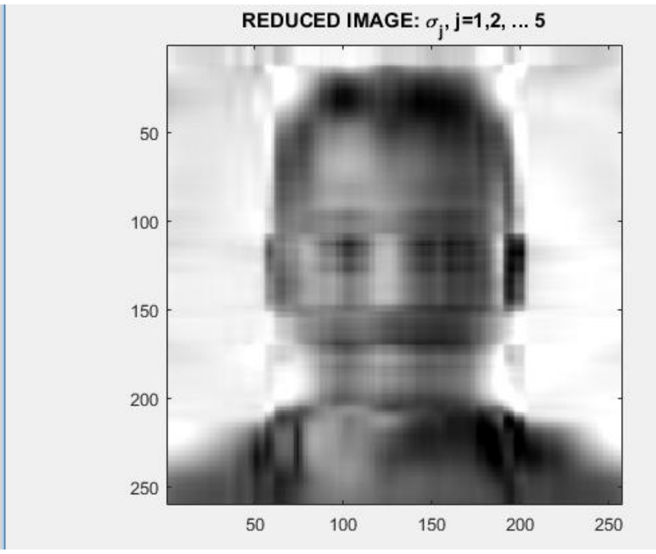
$$\begin{bmatrix} - & \underline{u}_1^T & - \\ - & \underline{u}_2^T & - \\ & \vdots & \\ - & \underline{u}_r^T & - \\ - & \underline{u}_{r+1}^T & - \\ & \vdots & \\ - & \underline{u}_m^T & - \end{bmatrix} A = \begin{bmatrix} - & \sigma_1 \underline{v}_1^T & - \\ - & \sigma_2 \underline{v}_2^T & - \\ & \vdots & \\ - & \sigma_r \underline{v}_1^T & - \\ - & \underline{0}^T & - \\ & \vdots & \\ - & \underline{0}^T & - \end{bmatrix}$$

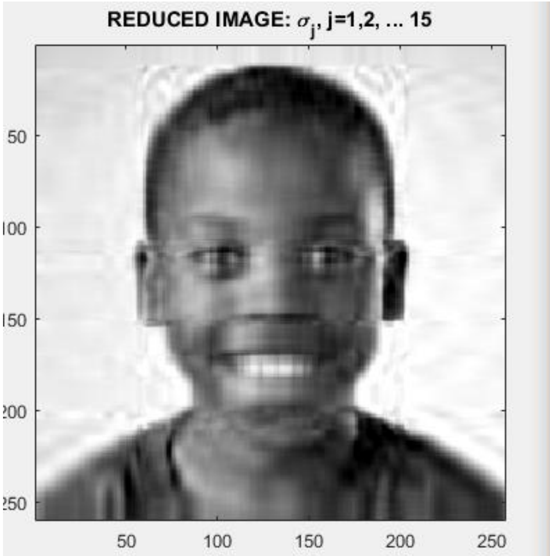
Last $m-r$ columns of U is a basis for $\mathcal{L}(A)$.

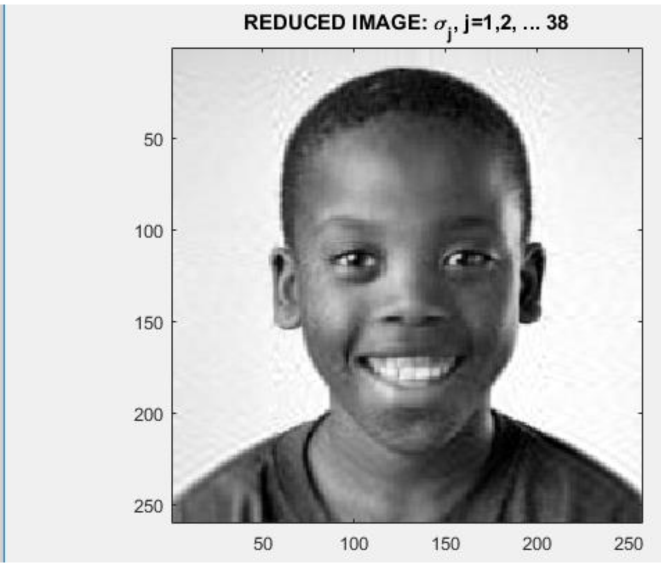
$\boxed{8}$

$$U = \Big[\ \underbrace{\ \mathcal{C}(A)\ }_{m}\ \Big|\ \underbrace{\ \mathcal{L}(A)\ }_{m-r}\ \Big] \qquad V^T = \begin{array}{r} r\left\{\right. \\ n-r\left\{\right. \end{array} \begin{bmatrix} R(A) \\ \hline N(A) \end{bmatrix}$$

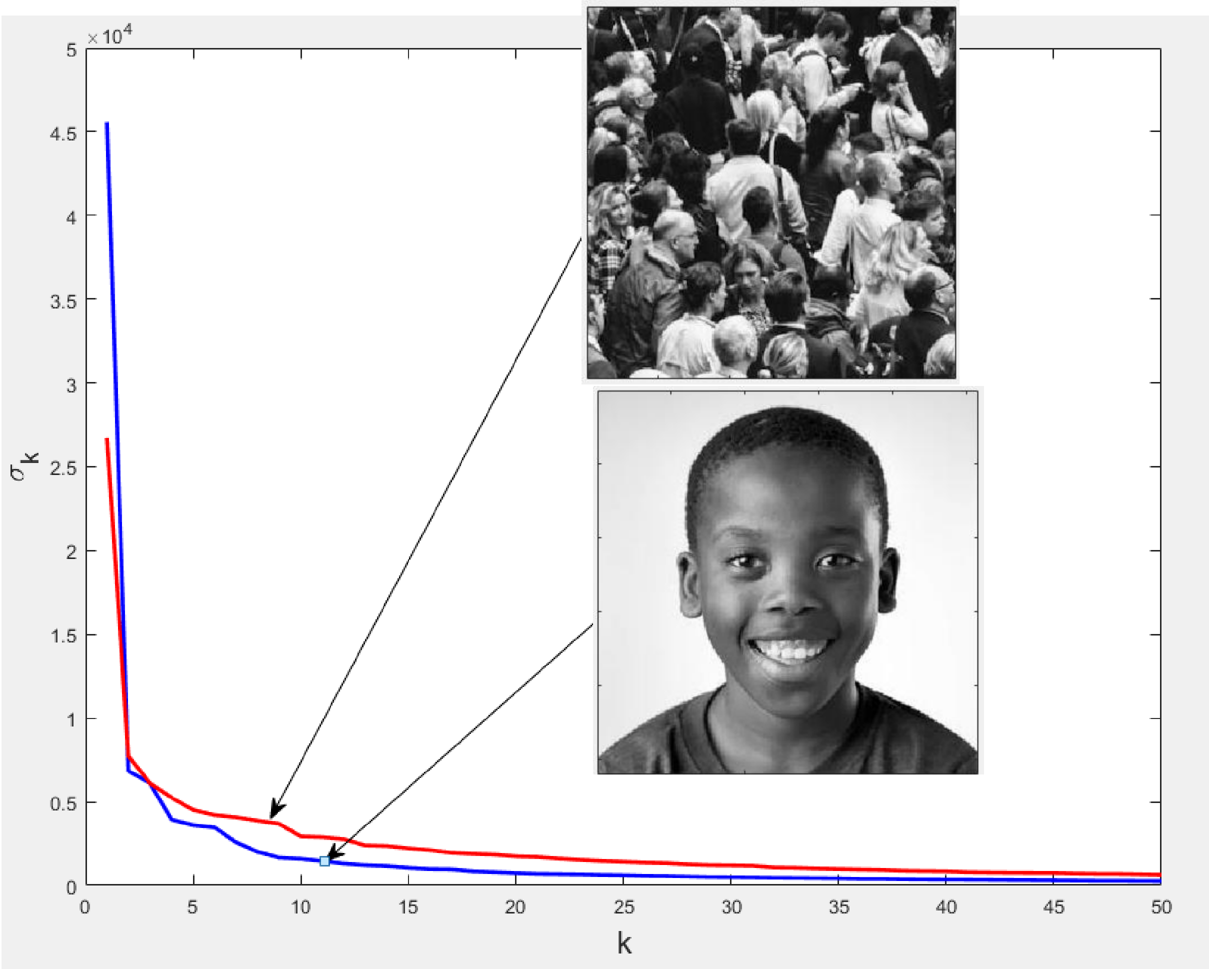








The image size is 260 x 257. Total number of pixels is 66820. The last reduction, with 38 singular values kept, needs storage for 260*38 + 257*38 + 38 values. That is 19684 values. The data reduction is 19684/66820 = 29.46%.



The blue graph plots the first 50 singular values of the bottom picture (one person's face) .

The red graph plots the first 50 singular values of the top picture (a crowd) .

The larger singular values for the higher values of k signify a more detailed image.