# AM214-2023: LECTURE 30

$\boxed{\text{LECTURE 30}}$ SYMMETRIC MAT'S AND DEGENARACY $\boxed{1}$

$A \in \mathbb{R}^{n \times n}$

Symmetric mat's $\quad A^T = A$

- Sym. mat's have real eval's
- Evec's of sym. mats are orthogonal, for evec's belonging to different eval's.

$\boxed{2}$

$2\times 2$ $\quad \begin{bmatrix} 0 & 22 \\ -6 & 11 \end{bmatrix}$ $\qquad A = \underset{\mathbb{R}}{S}\,\underset{\mathbb{R}}{\Lambda}\,\underset{\mathbb{R}}{S^{-1}}$

DIAGONALISATION OF SYM. MAT.

$A^T = A \qquad A x_1 = \lambda_1 x_1 \quad A x_2 = \lambda_2 x_2$

$$A\begin{bmatrix} x_1 & x_2 \end{bmatrix} = \underbrace{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}_{S}\begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$$

$\boxed{AS = S\Lambda}$ always.

If $S^{-1}$ exists, then

$$A = S\Lambda S^{-1}$$

$$S^{-1} = Q^{-1} = Q^T$$

$$Q\ \cancel{S} = \begin{bmatrix} x_1/\|x_1\| & x_2/\|x_2\| & x_3/\|x_3\| \end{bmatrix}$$

A sym. mat. can be diagonalised by $Q\Lambda Q^T$ $\boxed{3}$

<u>Example:</u>

$$K=\begin{bmatrix}1 & 8 & -2\\ 8 & 7 & -10\\ -2 & -10 & 10\end{bmatrix}. \quad x_1=\begin{bmatrix}-2\\2\\1\end{bmatrix} \text{ is an evec.}$$

$Kx_1=-6x_1$, Other eval's are $\alpha, \beta$.

$$-6\alpha\beta=-378 \qquad \alpha\beta=63$$

$$-6+\alpha+\beta=18 \qquad \alpha+\beta=24$$

$$\alpha=21,\ \beta=3$$

$$\downarrow$$

$$\underline{\lambda_1=-6,\ \lambda_2=21,\ \lambda_3=3}$$

$\boxed{4}$

$\underline{\lambda_2=21}$:

$$\begin{bmatrix}-20 & 8 & -2\\ 8 & -14 & -10\\ -2 & -10 & -11\end{bmatrix} \longrightarrow \qquad x_2=\begin{bmatrix}1\\2\\-2\end{bmatrix}$$


$$x_3=x_2\times x_1 \overset{\text{scaled}}{=} \begin{bmatrix}2\\1\\2\end{bmatrix}$$

<u>First normalize:</u>

$$Q=\frac{1}{3}\begin{bmatrix}-2 & 1 & 2\\ 2 & 2 & 1\\ 1 & -2 & 2\end{bmatrix}$$

$$\begin{matrix}-6 & 21 & 3\end{matrix}$$

5

A matrix with repeated eval's is called "DEGENARATE".

Sym. mats:

$$A=\begin{bmatrix} 8 & 4 & 2 \\ 4 & 8 & -2 \\ 2 & -2 & 11 \end{bmatrix} \qquad \underline{x}_1=\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} \qquad \lambda_1 = 3$$

$$\lambda_2 = 12, \quad \lambda_3 = 12$$

$$(A-12I) = \begin{bmatrix} -4 & 4 & 2 \\ 4 & -4 & -2 \\ 2 & -2 & -1 \end{bmatrix} \xrightarrow{LU} \begin{bmatrix} \boxed{-4} & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \mu \\ \kappa \end{bmatrix}$$

6

Get $\mathcal{N}(A-12I)$:

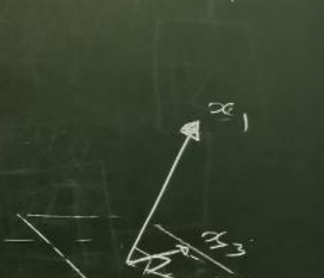
$$-4x_1 + 4\mu + 2\kappa = 0$$

$$-2x_1 + 2\mu + \kappa = 0$$

$$x_1 = \mu + \tfrac{1}{2}\kappa$$

$$\underline{x}_2 = \begin{bmatrix} \mu+\frac{1}{2}\kappa \\ \mu \\ \kappa \end{bmatrix} = \mu \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \kappa \begin{bmatrix} \frac{1}{2} \\ 0 \\ 1 \end{bmatrix}$$

$$S=\begin{bmatrix} -2 & 1 & \\ 2 & 1 & \\ 1 & 0 & \end{bmatrix}$$



$\boxed{7}$

$$x_3 = \begin{bmatrix} \mu + \frac{1}{2}\kappa \\ \mu \\ \kappa \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mu + \frac{1}{2}\kappa \\ \mu \\ \kappa \end{bmatrix} = 0$$

$$\mu + \tfrac{1}{2}\kappa + \mu = 0$$

$$2\mu = -\tfrac{1}{2}\kappa$$

$$4\mu = -\kappa \qquad 1 + \tfrac{1}{2}(-4)$$

$$\mu = 1$$

Choose

$$x_3 = \begin{bmatrix} -1 \\ 1 \\ -4 \end{bmatrix}$$

Unsymmetric degenerate matrices:

$$D = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \qquad x_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad x_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

v

$\boxed{8}$

$$DS = S\Lambda$$

$S$ is singular.


$D$ is degenerate, but it has too few independent evec's. Called

"DEFECTIVE"