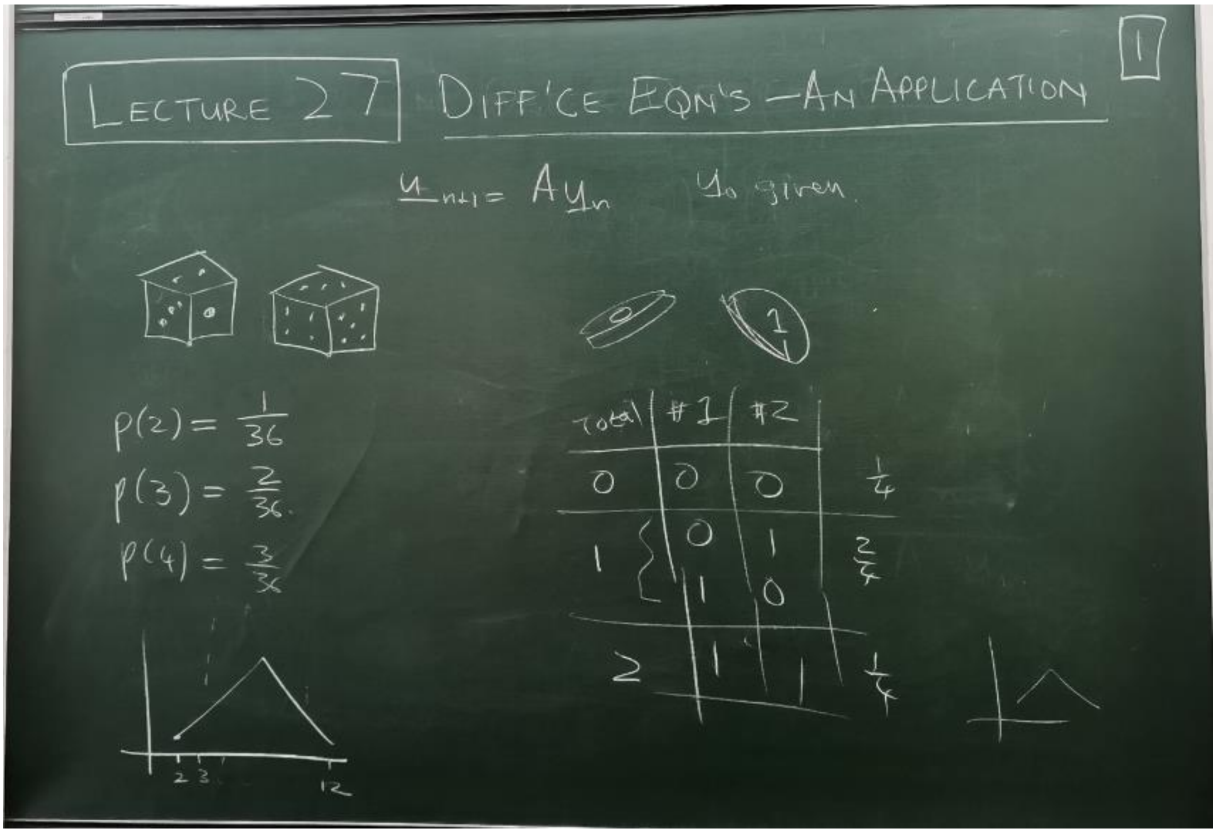
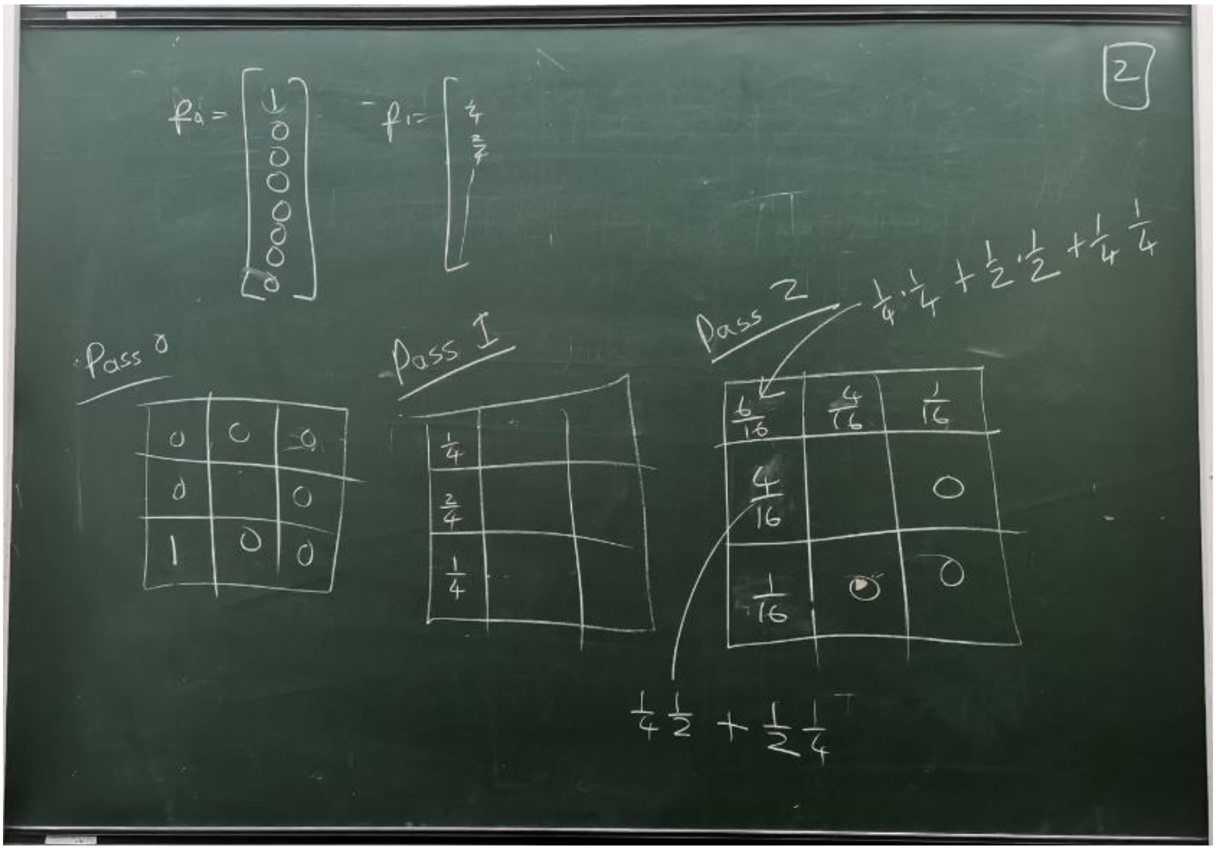
AM214-2023: LECTURE 27





$\boxed{3}$

No Jail

$$p_{n+1} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 \\ \vdots & & & & & & & \\ \vdots & & & & & & & \end{bmatrix} p_n$$

$$A$$

$$p_{n+1} = A p_n \qquad p_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} p_k &= A^k p_0 \\ &= S \Lambda^k S^{-1} p_0 \\ &= S \begin{bmatrix} 1^k & & & \\ & \text{frac}^k & & \\ & & \text{frac} & \\ & & & \ddots \end{bmatrix} S^{-1} p_0 \end{aligned}$$

$$S \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & & \end{bmatrix} S^{-1} p_0$$

$\boxed{4}$

$$p_{n+1} = \frac{1}{4} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cancel{1}0 & \cancel{2}0 & \cancel{1}0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{bmatrix} p_n$$

$$p_\infty = S \begin{bmatrix} 1 & 0 & 0 \\ 0 & & \\ \vdots & & \end{bmatrix} \underbrace{S^{-1} p_0}$$

$$= \begin{bmatrix} x_1 & x_2 & \cdots & x_8 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \vdots & & \end{bmatrix} \underline{y}$$

$$= \begin{bmatrix} \underline{x}_1 & \underline{0} & \underline{0} & \underline{0} & \cdots & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_\ell \end{bmatrix} = y_1 \underline{x}_1$$

$\boxed{5}$

$\boxed{6}$

DIF'L EQN'S

$$\dot{x} = \alpha x \longrightarrow x(t) = K e^{\alpha t}$$

$$x(t),\ y(t)$$

$$\dot{x} = ax + by$$
$$\dot{y} = cx + dy$$

$$x(0) = x_0$$
$$y(0) = y_0$$

$$\underline{\dot{u}} = \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{A} \underline{u} \qquad \underline{u}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$\underline{\dot{u}} = A\underline{u}$$

$$A = S \Lambda S^{-1}$$

$\boxed{7}$

$$\dot{\underline{u}} = S\Lambda \underbrace{S^{-1}\underline{u}}_{\underline{w}}$$

$$\underbrace{S^{-1}\dot{\underline{u}}}_{\dot{\underline{w}}} = \Lambda \underbrace{S^{-1}\underline{u}}_{\underline{w}}$$

$$\dot{\underline{w}} = \Lambda \underline{w}$$

$$\begin{bmatrix} \dot w_1 \\ \dot w_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\dot w_1 = \lambda_1 w_1 \longrightarrow w_1(t) = K_1 e^{\lambda_1 t}$$

$$\dot w_2 = \lambda_2 w_2 \longrightarrow w_2(t) = \{S^{-1}\underline{u}(0)\}_2 e^{\lambda_2 t}$$

$$\{S^{-1}\underline{u}(0)\}_1 = K_1 e^0$$

$$\underline{w}(t) = S^{-1}\underline{u}(t)$$

$$\underline{u}(t) = S\,\underline{w}(t)$$

$$\dot{\underline{u}}(t) = \underbrace{\cancel{\dot S}}_{0}\,\underline{w} + S\dot{\underline{w}}$$

$$\dot{\underline{u}} = S\dot{\underline{w}}$$

$$\dot{\underline{w}} = S^{-1}\dot{\underline{u}}$$

$$\underline{w}(0) = S^{-1}\underline{u}(0)$$

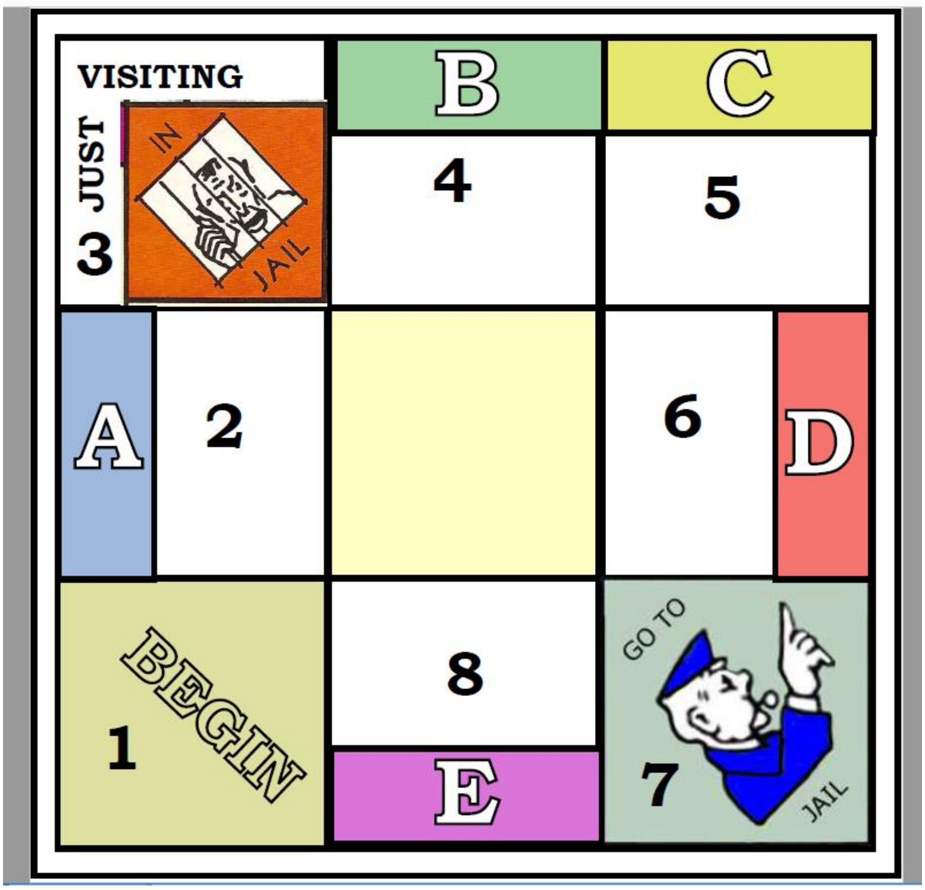
$$\{S^{-1}\underline{u}(0)\}_1$$

$\boxed{8}$

$$\underline{w}(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix} S^{-1}\underline{u}(0)$$

$$\underline{u}(t) = S\,\underline{w}(t) = S \underbrace{\begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}}_{e^{\Lambda t}} S^{-1}\underline{u}(0)$$

$$\boxed{\underline{u}(t) = S e^{(\Lambda t)} S^{-1}\underline{u}(0)}$$


VISITING
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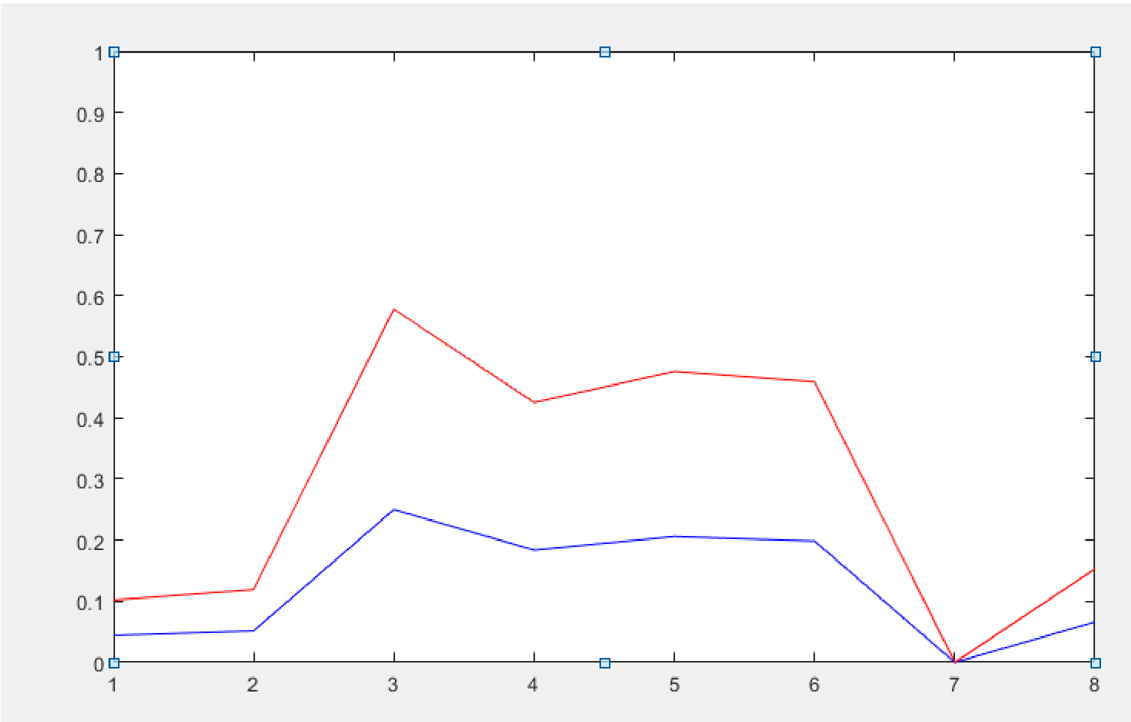
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