AM214-2023: LECTURE 26

$\boxed{\text{LECTURE 26}}$ 2ND ORDER DIFF'CE EQN'S $\boxed{1}$

$$x_{n+1} = a x_n + b y_n \qquad x_0 = \text{given}$$
$$y_{n+1} = c x_n + d y_n \qquad y_0 = \text{given.}$$

$$\underline{u}_{n+1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \underline{u}_n$$

$$\boxed{\underline{u}_{n+1} = A\underline{u}_n} \qquad \underline{u}_0 \ \cancel{\text{given}}.$$

$$\underline{u}_0 = A^{-1}\underline{u}_1$$

$\underline{u}_1$ given.

$$\underline{u}_k = A^{k-1}\underline{u}_1$$

solution

$$\underline{u}_k = A^k \underline{u}_0 \qquad A = S\Lambda S^{-1}$$
$$= S\Lambda^k S^{-1}\underline{u}_0$$

2ND ORDER DIFF'CE EQN'S $\boxed{2}$

$$T_{n+1} = a T_n + b T_{n-1} \qquad T_0 = \text{given}$$
$$T_1 = \text{given.}$$

Convert to 2×2 first order eqn's.

Let $S_n = T_{n-1}$

$$T_{n+1} = a T_n + b S_n.$$
$$S_{n+1} = T_n$$

$$\begin{pmatrix} T_{n+1} \\ S_{n+1} \end{pmatrix} = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} T_n \\ S_n \end{bmatrix} \qquad T_0 = 0, \quad T_1 = 1$$

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$$\uparrow \qquad\qquad \uparrow$$
$$\underline{y}_{n+1} \qquad\qquad \underline{y}_n$$

$$y_0 = \begin{bmatrix} T_0 \\ S_0 \end{bmatrix} = \begin{bmatrix} T_0 \\ T_{-1} \end{bmatrix}$$

$$\underline{y}_1 = \begin{bmatrix} T_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_0 \end{bmatrix}$$

Example:

$$T_{n+1} = 2T_n + 3T_{n-1}$$

$$T_0 = 0$$
$$T_1 = 1$$

T_0	T_1	T_2	T_3	T_4
0	1	2	7	20

$$T_{n+1} = 2T_n + 3T_{n-1}$$
$$T_n = T_n$$

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$$\begin{bmatrix} T_{n+1} \\ T_n \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} T_n \\ T_{n-1} \end{bmatrix}}_{\underline{y}_n} \qquad \underbrace{\begin{bmatrix} T_1 \\ T_0 \end{bmatrix}}_{\underline{y}_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underline{y}_{n+1} = A\,\underline{y}_n$$

$$A = S\Lambda S^{-1}$$

$$\lambda^2 - 2\lambda + (-3) = 0, \quad \lambda_1 = -1, \quad \lambda_2 = +3$$

$$(-1-\lambda)(3-\lambda)$$

$\underline{\lambda_1 = -1}$:

$$\begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \mu \end{bmatrix} = \underline{0}$$

$$x_1 = -\mu$$

$$\underline{x}_1 = \begin{bmatrix} -\mu \\ \mu \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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$\underline{\lambda_2 = 3}$

$$\begin{bmatrix}-1 & 3\\ 1 & -3\end{bmatrix} \longrightarrow \begin{bmatrix}-1 & 3\\ 0 & 0\end{bmatrix}\begin{pmatrix}x_1\\ \mu\end{pmatrix} = \underline{0}$$

$$-x_1 = -3\mu \qquad x_1 = 3\mu$$

$$\underline{x}_2 = \begin{bmatrix}3\mu\\ \mu\end{bmatrix} = \begin{bmatrix}3\\ 1\end{bmatrix}$$

$$\Lambda = \begin{bmatrix}3 & 0\\ 0 & -1\end{bmatrix} \qquad S = \begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix}, \quad S^{-1} = \frac{1}{4}\begin{bmatrix}1 & 1\\ -1 & 3\end{bmatrix}$$

$$\underline{u}_k = S\Lambda^k S^{-1}\underline{u}_0$$

$$= S\Lambda^{k-1}S^{-1}\underline{u}_1$$

$$=$$

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$$\underline{u}_k = \frac{1}{4}\underbrace{\begin{bmatrix}3 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}3^{k-1} & 0\\ 0 & (-1)^{k-1}\end{bmatrix}}\underbrace{\begin{bmatrix}1 & 1\\ -1 & 3\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}}$$

$$= \frac{1}{4}\begin{bmatrix}3^k & (-1)^k\\ 3^{k-1} & (-1)^{k-1}\end{bmatrix}\begin{bmatrix}1\\ -1\end{bmatrix}$$

$$\begin{pmatrix}T_k\\ T_{k-1}\end{pmatrix} = \frac{1}{4}\begin{bmatrix}3^k - (-1)^k\\ 3^{k-1} - (-1)^{k-1}\end{bmatrix}$$

$$\boxed{T_k = \frac{3^k - (-1)^k}{4}} \qquad T_4 = \frac{81-1}{4} = 20$$

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Example of diff'ce eqns

$$2H_2 + O_2 \rightarrow 2H_2O$$

$H\cdot$ $\quad$ $OH\cdot$

$$O\cdot + H_2 \rightarrow OH\cdot + H\cdot$$

$$OH\cdot + H_2 \rightarrow H\cdot + H_2O$$

$$H\cdot + O_2 \rightarrow OH\cdot + O\cdot$$

$\boxed{8}$

O_n = number of $O\cdot$ radicals

OH_n = " " $OH\cdot$ "

H_n = " " $H\cdot$

$$O_{n+1} = H_n$$

$$H_{n+1} = O_n + OH_n$$

$$OH_{n+1} = H_n + O_n$$

$$\begin{bmatrix} O_{n+1} \\ OH_{n+1} \\ H_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} O_n \\ OH_n \\ H_n \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Get eigenvalues. $\quad$ 1.618

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DIFFERENTIAL EQN'S

$x(t)$

$\dot{x} = \alpha x$ α is given, $x(0) =$ given.

$N(t) =$ number of moles U_{235}.

$\dot{N} = \alpha N$

$\uparrow$

$\alpha < 0$

$$\frac{dx}{dt} = \alpha x$$

$$\int \frac{dx}{x} = \int \alpha \, dt$$

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$$\ln x = \alpha t + C$$

$$x = e^{C+\alpha t} = e^{C} e^{\alpha t}$$

$$\boxed{x(t) = K e^{\alpha t}}$$

SYSTEMS OF 1^{ST} ORDER DIF'L EQN'S.

$\dot{x} = ax + by$ $x(0) = x_0$

$\dot{y} = cx + dy$ $y(0) = y_0$

$$\underline{u}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad \dot{\underline{u}}(t) = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix}$$

$$\dot{\underline{u}} = A\underline{u}$$