

LECTURE 20

1

Q is an $n \times n$ orthogonal matrix

$$Q^T Q = I$$

Use: Solve $A \underline{x} = \underline{b}$
 $QR \underline{x} = \underline{b}$,

$$R \underline{x} = Q^T \underline{b}$$

↑ upper triangular

$$A = \begin{bmatrix} a & b & c \end{bmatrix}$$

Step 1

\underline{a}

\rightarrow

$$\underline{q}_1 = \frac{\underline{a}}{\|\underline{a}\|}$$

$$Q = \begin{bmatrix} | & | \\ \underline{q}_1 & \underline{q}_2 \\ | & | \end{bmatrix}$$

Step 2:

$$\underline{b}' = \underline{b} - (\underline{q}_1 \underline{q}_1^T) \underline{b}$$

\rightarrow Subtract all along \underline{q}_1 .
What remains is
orth. to \underline{q}_1 .

$$\underline{q}_2 = \frac{\underline{b}'}{\|\underline{b}'\|}$$

$$c' = c - (q_1 q_1^T) c - (q_2 q_2^T) c$$

$$q_3 = \frac{c'}{\|c'\|}$$

What about R.

$$A = QR$$

$$Q^T A = R$$

$$Q^T A = \begin{bmatrix} - & q_1^T & 1 \\ - & q_2^T & 1 \\ - & q_3^T & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 10 \\ 15 \end{bmatrix} = \begin{bmatrix} \cancel{q_1^T a} & \cancel{q_1^T b} & q_1^T c \\ \cancel{q_2^T a} & \cancel{q_2^T b} & q_2^T c \\ \cancel{q_3^T a} & \cancel{q_3^T b} & q_3^T c \end{bmatrix}$$

$\|a\|$



$$\underline{a}^T \underline{a}$$

$$\underline{q}_1 = \frac{\underline{a}}{\|\underline{a}\|}$$

$$\underline{q}_1^T \underline{a} = \left(\frac{\underline{a}}{\|\underline{a}\|} \right)^T \underline{a}$$

$$\underline{q}_1^T \underline{a} = \sqrt{\underline{a}^T \underline{a}}$$

$$= \frac{\underline{a}^T \underline{a}}{\sqrt{\underline{a}^T \underline{a}}} = \sqrt{\underline{a}^T \underline{a}}$$

[3]

Step 2

$$\underline{b}' = \underline{b} - (\underline{q}_1^T \underline{b}) \underline{q}_1$$

$$= \underline{b} - \underline{q}_1 (\underline{q}_1^T \underline{b})$$

Step 3

$$\underline{c}' = \underline{c} - \underline{q}_1 (\underline{q}_1^T \underline{c}) - \underline{q}_2 (\underline{q}_2^T \underline{c})$$

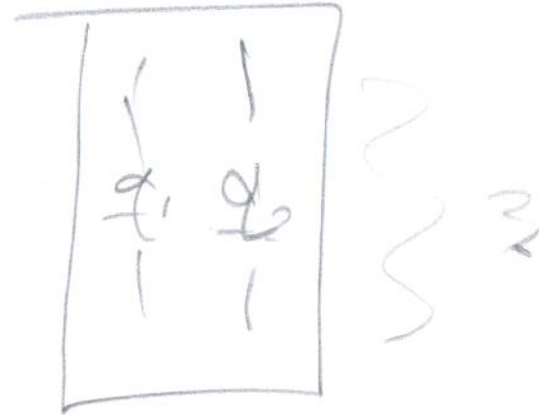
$$R = \begin{bmatrix} \|a\| & q_1^T b & q_1^T c \\ 0 & \|b\| & q_2^T c \\ 0 & 0 & \|c\| \end{bmatrix}$$

← Step 2
Step 3

4

Reduced QR

\bar{Q} is $m \times m$



\bar{Q} is $m \times n$, $m > n$.

$\bar{Q}^T \bar{Q} = I$

A diagram illustrating the equation $\bar{Q}^T \bar{Q} = I$. It shows a horizontal rectangle on the left, followed by a vertical rectangle, an equals sign, and a small square on the right.

but

$\bar{Q} \bar{Q}^T \neq I$

A diagram illustrating the equation $\bar{Q} \bar{Q}^T \neq I$. It shows a vertical rectangle on the left, followed by a horizontal rectangle, an equals sign, and a larger square on the right.