LECTURE 20

Q is an $n \times n$ orthogonal matrix

$Q^T Q = I$

Use: Solve $A\underline{x} = \underline{b}$

$QR\underline{x} = \underline{b}$, $\quad R\underline{x} = Q^T\underline{b}$

$\uparrow$ upper triang.

$A = [\underline{a} \ \ \underline{b} \ \ \underline{c}]$

Step 1

$\underline{a} \longrightarrow q_1 = \dfrac{\underline{a}}{\|\underline{a}\|}$

$$Q = \begin{bmatrix} | & | \\ q_1 & q_2 \\ | & | \end{bmatrix}$$

Step 2:

$\underline{b}' = \underline{b} - (q_1 q_1^T)\underline{b}$

$\rightarrow$ Subtract all along q_1.
What remains is orth. to q_1.

$q_2 = \dfrac{\underline{b}'}{\|\underline{b}'\|}$

$$\underline{c}' = \underline{c} - (\underline{q}_1 \underline{q}_1^T)\underline{c} - (\underline{q}_2 \underline{q}_2^T)\underline{c}$$

$$\underline{q}_3 = \frac{\underline{c}'}{\|\underline{c}'\|}$$

What about R.

$$A = QR$$

$$Q^T A = R$$

$$Q^T A = \begin{bmatrix} - \underline{q}_1^T - \\ - \underline{q}_2^T - \\ - \underline{q}_3^T - \end{bmatrix} \begin{bmatrix} | & | & | \\ \underline{a} & \underline{b} & \underline{c} \\ | & | & | \end{bmatrix} = \begin{bmatrix} \cancel{\underline{q}_1^T \underline{a}} & \underline{q}_1^T \underline{b} & \underline{q}_1^T \underline{c} \\ \cancel{\underline{q}_2^T \underline{a}} & \underline{q}_2^T \underline{b} & \underline{q}_2^T \underline{c} \\ \cancel{\underline{q}_3^T \underline{a}} & \cancel{\underline{q}_3^T \underline{b}} & \underline{q}_2^T \underline{c} \end{bmatrix}$$

$\|\underline{a}\|$

0

0 0

$\boxed{3}$

$$(\underline{a}^T\underline{a}) \longrightarrow q_1 = \frac{\underline{a}}{\sqrt{\|\underline{a}^T\underline{a}\|}}$$

$$q_1^T\underline{a} = \left(\frac{\underline{a}}{\|\underline{a}^T\underline{a}\|}\right)^T \underline{a}$$

$$= \frac{\underline{a}^T\underline{a}}{\sqrt{\underline{a}^T\underline{a}}} = \sqrt{\underline{a}^T\underline{a}}$$

$$q_1^T\underline{a} = \sqrt{\underline{a}^T\underline{a}}$$

Step 2

$$\underline{b}' = \underline{b} - (q(q_1^T)\underline{b})$$

$$= \underline{b} - q_1(q_1^T\underline{b})$$

Step 3

$$\underline{c}' = \underline{c} - q_1(q_1^T\underline{c}) - q_2(q_2^T\underline{c})$$

Step 2

Step 3

$$R = \begin{bmatrix} \|\underline{a}\| & q_1^T \underline{b} & q_1^T \underline{c} \\ 0 & \|\underline{b}'\| & q_2^T \underline{c} \\ 0 & 0 & \|\underline{c}'\| \end{bmatrix}$$

Reduced QR

$\bar{Q}$ is $n \times m$

$\bar{Q}$ is $m \times n$, $m > n$.

$\bar{Q}^T \bar{Q} = I$

but

$\bar{Q} \bar{Q}^T \neq I$