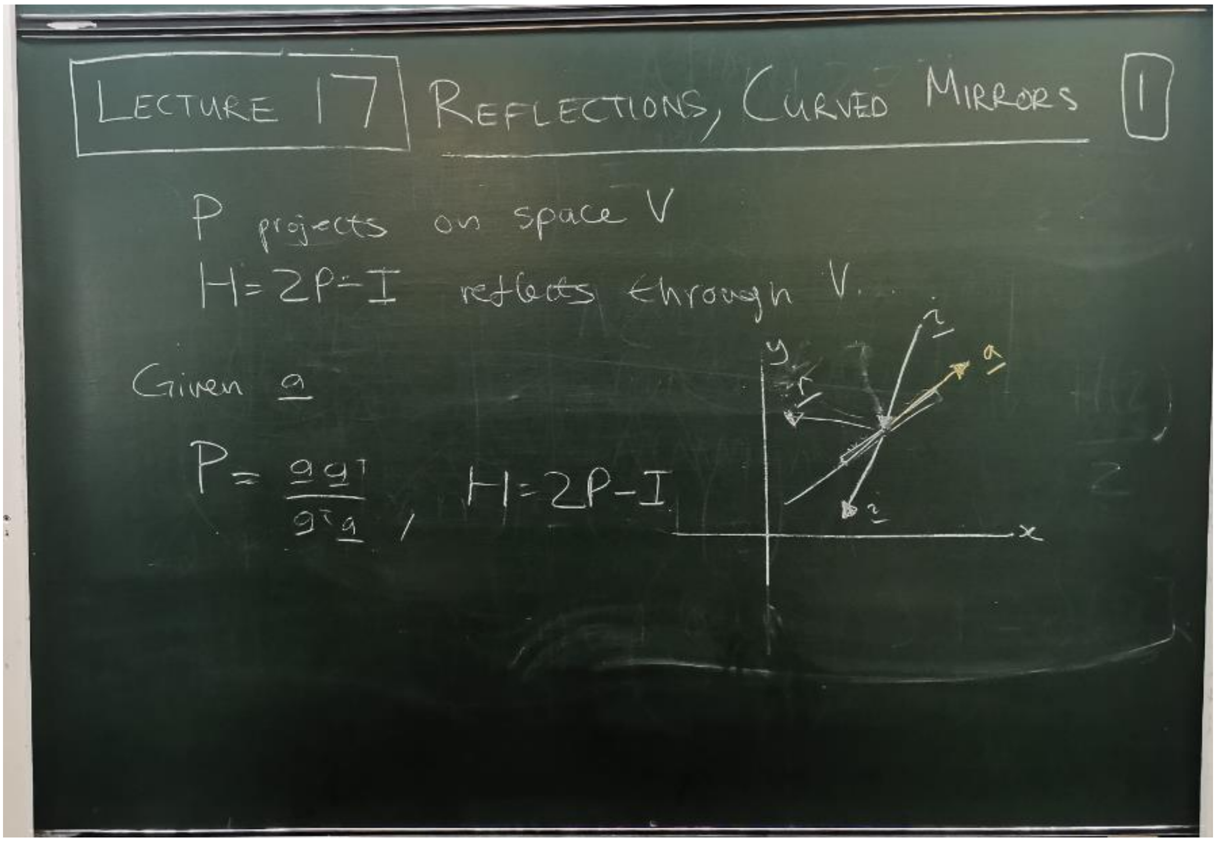
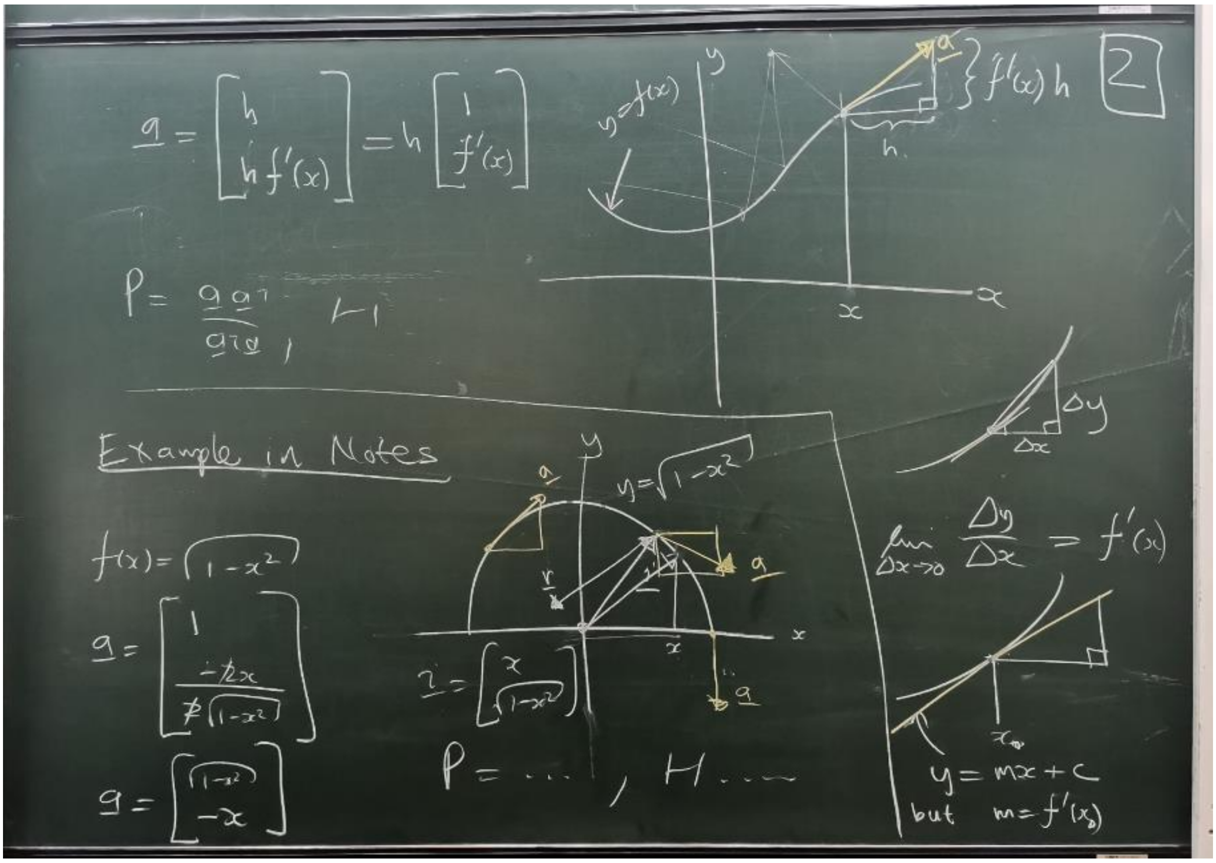
AM214-2023: LECTURE 17

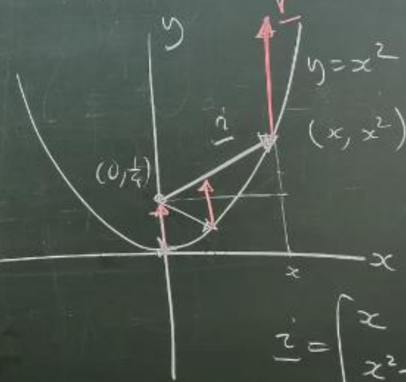




$\boxed{3}$

Example: Show that light from $(0, \frac{1}{4})$ falling on parabola $y = x^2$, reflects vertically.

$$\underline{g} = \begin{bmatrix} 1 \\ 2x \end{bmatrix}$$



$$\underline{i} = \begin{bmatrix} x \\ x^2 - \frac{1}{4} \end{bmatrix}$$

$$P = \frac{\underline{g}\,\underline{g}^T}{\underline{g}^T \underline{g}} = \frac{\begin{bmatrix} 1 \\ 2x \end{bmatrix} \begin{bmatrix} 1 & 2x \end{bmatrix}}{\begin{bmatrix} 1 & 2x \end{bmatrix} \begin{bmatrix} 1 \\ 2x \end{bmatrix}}$$

$$P = \frac{1}{1+4x^2} \begin{bmatrix} 1 & 2x \\ 2x & 4x^2 \end{bmatrix}$$

$\boxed{4}$

$$H = 2P - I$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 2 & 4x \\ 4x & 8x^2 \end{bmatrix} - \frac{1}{1+4x^2} \begin{bmatrix} 1+4x^2 & 0 \\ 0 & 1+4x^2 \end{bmatrix}$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 1-4x^2 & 4x \\ 4x & 8x^2 - 1 - 4x^2 \end{bmatrix}$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 1-4x^2 & 4x \\ 4x & 4x^2 - 1 \end{bmatrix}$$

$$\underline{r} = H \underline{i}$$

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$$= \frac{1}{1+4x^2}\begin{bmatrix} 1-4x^2 & 4x \\ 4x & 4x^2-1 \end{bmatrix}\begin{bmatrix} x \\ x^2-\frac{1}{4} \end{bmatrix}$$

$$= \frac{1}{1+4x^2}\begin{bmatrix} x-4x^3 + 4x^3 - x \\ 4x^2 + (4x^2-1)(x^2-\frac{1}{4}) \end{bmatrix} = \begin{bmatrix} 0 \\ \text{something} \end{bmatrix}$$

Invert:

$$\underline{i} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\underline{r} = H\underline{v} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$\to \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$y = mx + c.$

$\uparrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$m = \frac{r_2}{r_1}$

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$$y = b\sqrt{1-\frac{x^2}{a^2}}$$

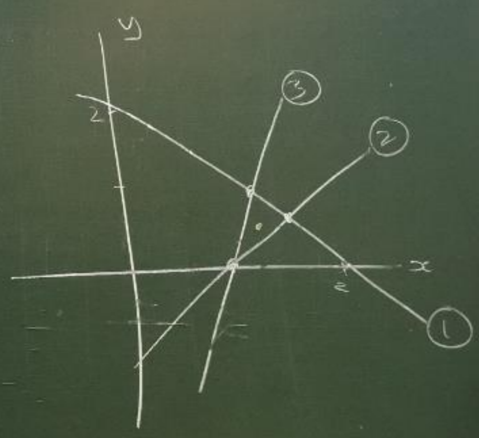
CHAPTER 8: LEAST SQUARES SOL.

$\boxed{7}$

A is $m \times n$, $m > n$, $\quad A\underline{x} \overset{?}{=} \underline{b}$

$$\begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -\frac{1}{3} \end{bmatrix}$$



$\boxed{8}$

$A\underline{x} \neq \underline{b}$ because $\underline{b} \notin \mathcal{C}(A)$

$$\begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = x \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \underline{b}$$