

AM214-2023: LECTURE 17

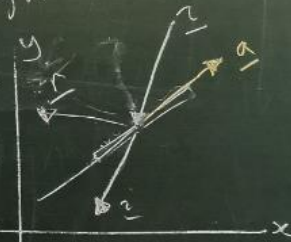
LECTURE 17 REFLECTIONS, CURVED MIRRORS [1]

P projects on space V

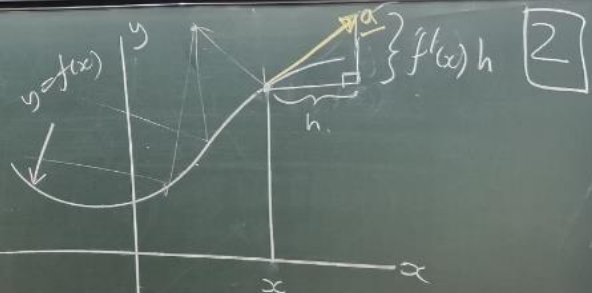
$H = 2P - I$ reflects through V

Given \underline{a}

$$P = \frac{\underline{a}\underline{a}^T}{\underline{a}^T\underline{a}}, \quad H = 2P - I$$



$$\underline{a} = \begin{bmatrix} h \\ hf'(x) \end{bmatrix} = h \begin{bmatrix} 1 \\ f'(x) \end{bmatrix}$$



$$P = \frac{\underline{a}\underline{a}^T}{\underline{a}^T\underline{a}}, \quad H = 2P - I$$

Example in Notes

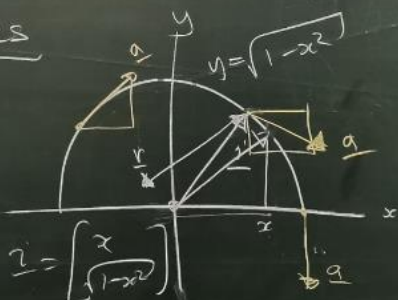
$$f(x) = \sqrt{1-x^2}$$

$$\underline{a} = \begin{bmatrix} 1 \\ -2x \\ \sqrt{1-x^2} \end{bmatrix}$$

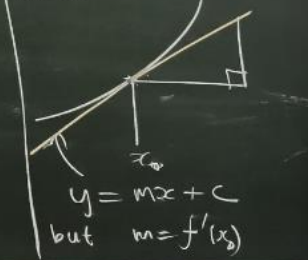
$$\underline{a} = \begin{bmatrix} \sqrt{1-x^2} \\ -x \end{bmatrix}$$

$$\underline{z} = \begin{bmatrix} x \\ \sqrt{1-x^2} \end{bmatrix}$$

$$P = \dots, \quad H = \dots$$



$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = f'(x)$$

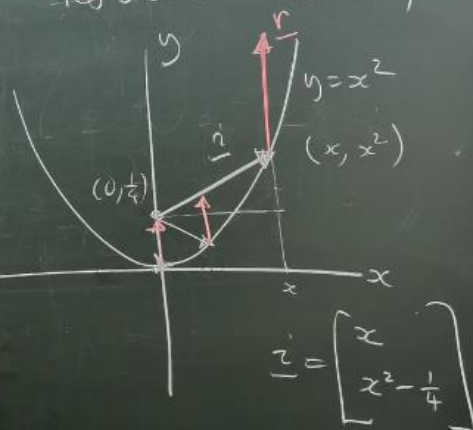


Example: Show that light from $(0, \frac{1}{4})$ 3
 falling on parabola $y = x^2$, reflects vertically.

$$g = \begin{bmatrix} 1 \\ 2x \end{bmatrix}$$

$$P = \frac{g g^T}{g^T g} = \frac{\begin{bmatrix} 1 \\ 2x \end{bmatrix} \begin{bmatrix} 1 & 2x \end{bmatrix}}{\begin{bmatrix} 1 & 2x \end{bmatrix} \begin{bmatrix} 1 \\ 2x \end{bmatrix}}$$

$$P = \frac{1}{1+4x^2} \begin{bmatrix} 1 & 2x \\ 2x & 4x^2 \end{bmatrix}$$



$$H = 2P - I$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 2 & 4x \\ 4x & 8x^2 \end{bmatrix} - \frac{1}{1+4x^2} \begin{bmatrix} 1+4x^2 & 0 \\ 0 & 1+4x^2 \end{bmatrix}$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 1-4x^2 & 4x \\ 4x & 8x^2-1-4x^2 \end{bmatrix}$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 1-4x^2 & 4x \\ 4x & 4x^2-1 \end{bmatrix}$$

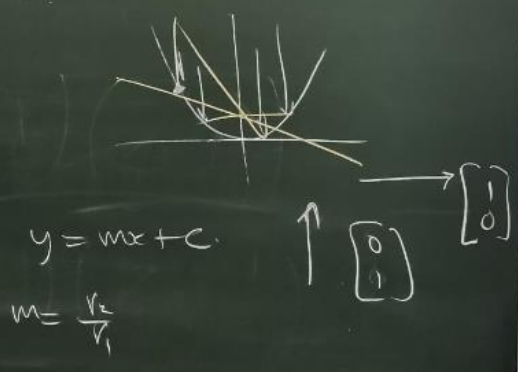
$$\underline{r} = H \underline{z}$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} 1-4x^2 & 4x \\ 4x & 4x^2-1 \end{bmatrix} \begin{bmatrix} x \\ x^2 - \frac{1}{4} \end{bmatrix} \quad [5]$$

$$= \frac{1}{1+4x^2} \begin{bmatrix} x-4x^3 + 4x^3 - x \\ 4x^2 + (4x^2-1)(x^2 - \frac{1}{4}) \end{bmatrix} = \begin{bmatrix} 0 \\ \text{something} \end{bmatrix}$$

Invent: $\vec{r} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\vec{r} = t\vec{v}_1 + u\vec{v}_2 = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



$$y = b \sqrt{1 - \frac{x^2}{a^2}}$$

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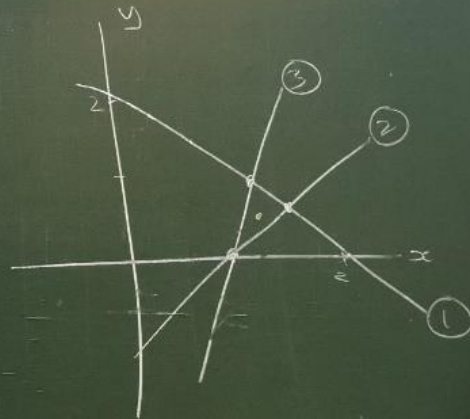
CHAPTER 8: LEAST SQUARES SOL.

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A is $m \times n$, $m > n$, $A \vec{x} = \vec{b}$

$$\begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$$



$A \vec{x} \neq \vec{b}$ because $\vec{b} \notin \text{Col}(A)$

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$$\begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \vec{b}$$

