AM214-2023: LECTURE 16

LECTURE 16 — REFLECTIONS IN 3D

1

SUMMARY OF PROJECTIONS:

$$A = \begin{bmatrix} | & | & & | \\ \underline{a}_1 & \underline{a}_2 & \cdots & \underline{a}_r \\ | & | & & | \end{bmatrix}$$ ← portrait.

$$P = A(A^TA)^{-1}A^T$$

$\underline{a}_j \in \mathbb{R}^m$, $r < m$

- P^{-1} does not exist.
- $P^T = P$
- $P^2 = P$
- $I - P$ is also a proj. matrix

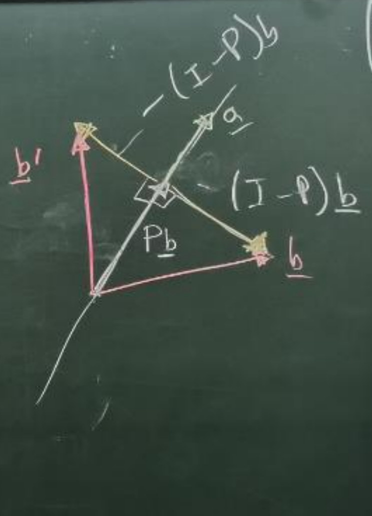
2

REFL. THROUGH A LINE

P projects on $\underline{a}$

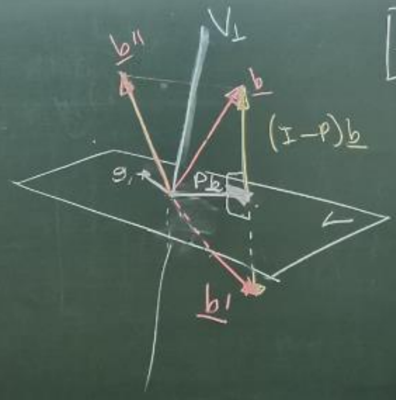
$$\underline{b}' = P\underline{b} - (I-P)\underline{b}$$
$$= \underbrace{(2P - I)}_{H}\underline{b}$$

$$\boxed{H_{\text{line}} = 2P_{\text{line}} - I}$$



3

$$P = A(A^TA)^{-1}A^T$$



$$\underline{b}' = P\underline{b} - (I-P)\underline{b}$$

$$= \underbrace{(2P_{plane} - I)}_{H}\underline{b}$$

$$\boxed{H_{plane} = 2P_{plane} - I}$$

General: If P projects on V, then

$H = 2P - I$ reflects through V.

4

PROPERTIES OF H:

- H^{-1} exists.
- $H^T = H$
- $H^2 = I$

$$\left[\begin{aligned} H^2 &= (2P-I)(2P-I) \\ &= 4P^2 - 2P - 2P + I \\ &= I \end{aligned}\right.$$

V, $V_\perp$

- If H_V reflects through V,

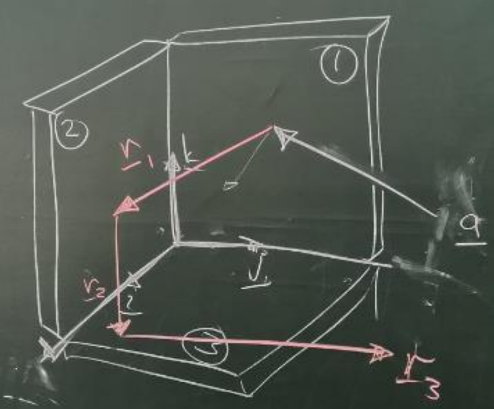
then $\boxed{\begin{aligned} H_{V_\perp} &= -(2P-I) \\ &= -H_V \end{aligned}}$ reflects throug $V_\perp$

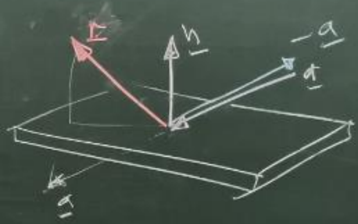
$H_V = 2P - I$

$H_{V_\perp} = 2(I-P) - I = 2I - 2P - I = I - 2P = -(2P-I)$

$\boxed{5}$

RETRO-REFLECTOR





$$H_{1} = 2\,\frac{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\end{bmatrix}}{\underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}_{1}} - I = 2\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$$

$\boxed{6}$

$$\underline{r}_1 = H_{1}(-\underline{a})$$

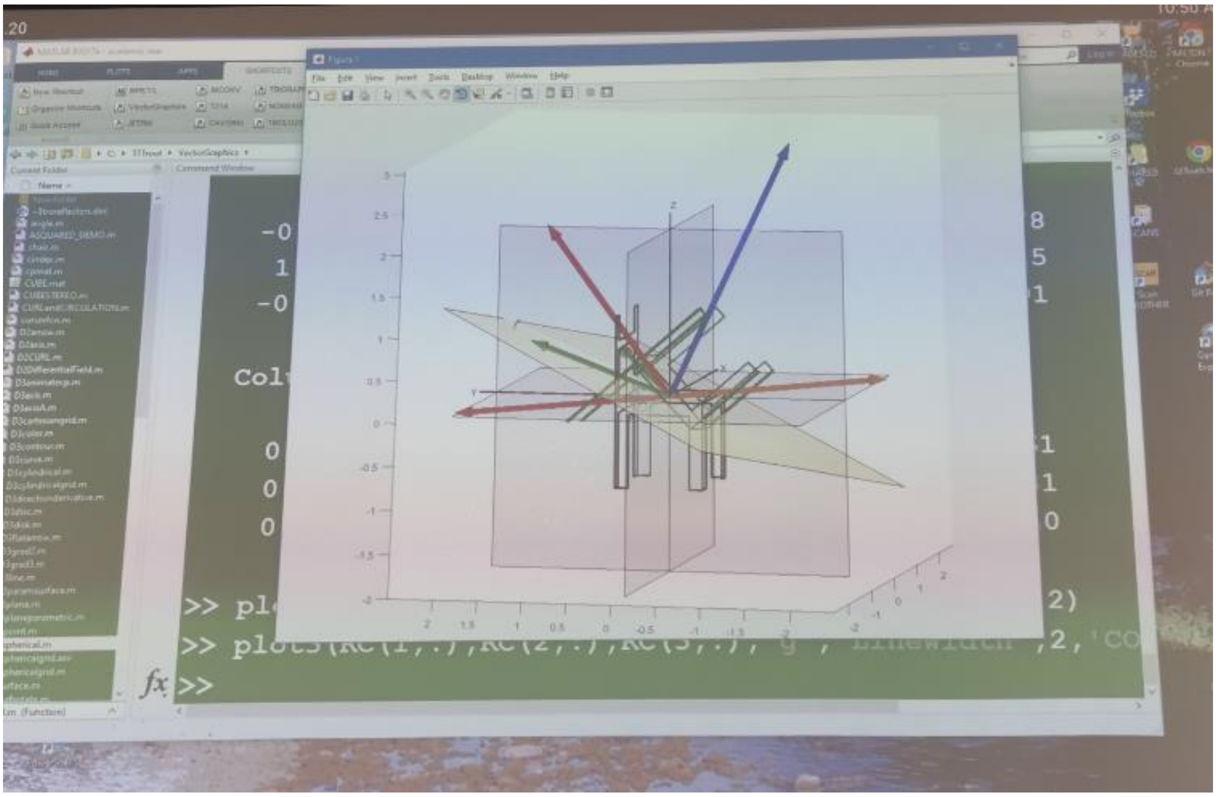
$$H_{2} = \begin{bmatrix}-1 & & \\ & 1 & \\ & & -1\end{bmatrix}$$

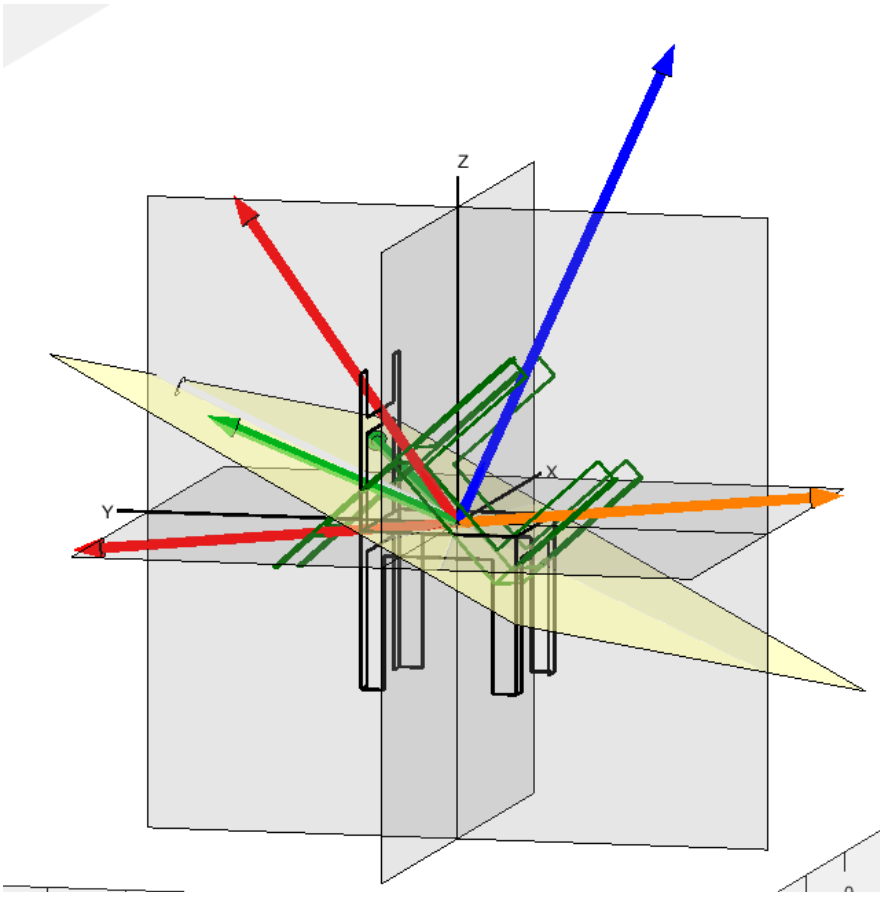
$$H_{3} = \begin{bmatrix}-1 & & \\ & -1 & \\ & & 1\end{bmatrix}$$

$$\underline{r}_2 = H_{2}(-\underline{r}_1)$$
$$= H_{2}(-H_{1}(-\underline{a}))$$
$$= H_{2}H_{1}\underline{a}$$

$$\underline{r}_3 = H_{3}(-\underline{r}_2) = -H_{3}H_{2}H_{1}\underline{a}$$

$$\underline{r}_3 = -I\underline{a} = -\underline{a}$$





Z
X
Y

TWO OF THE FIVE LUNAR RETRO REFLECTORS