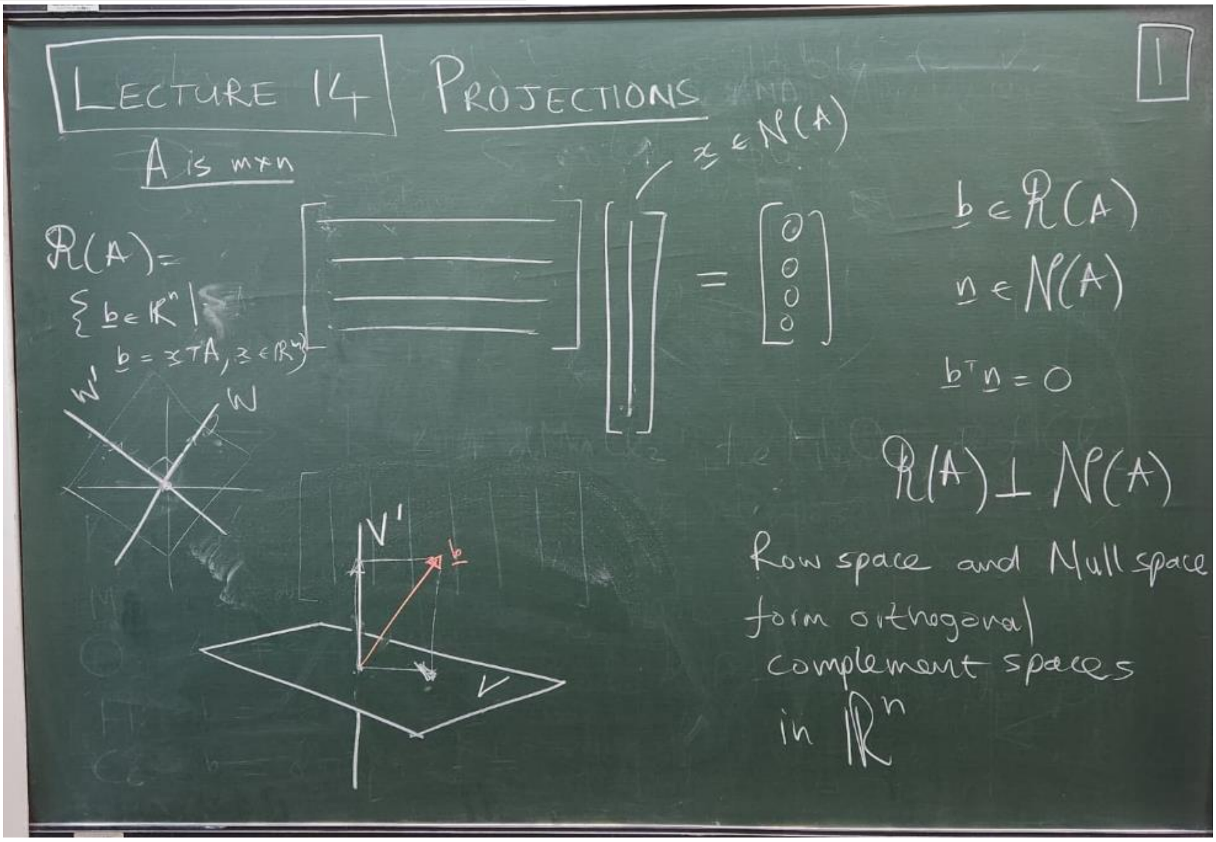
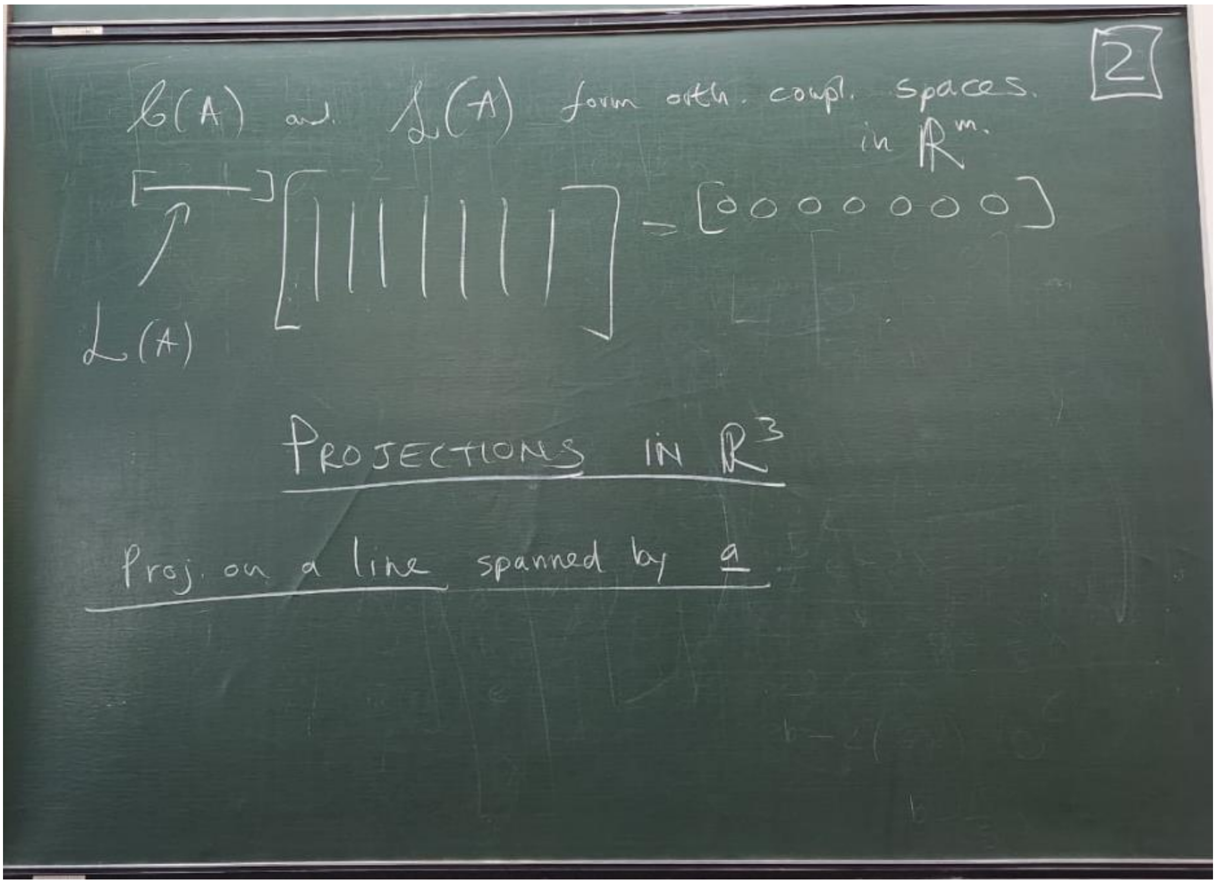
AM214-2023: LECTURE 14





$\boxed{3}$

About $\underline{p}$

① $\underline{p} = \lambda \underline{a}$

② $\underline{p} + \underline{q} = \underline{b}$

③ $\underline{a}^T \underline{q} = 0$

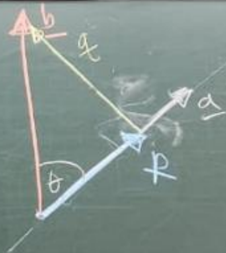
Using LA

②: $\underline{q} = \underline{b} - \underline{p}$

①: $\underline{q} = \underline{b} - \lambda \underline{a}$

③: $\underline{a}^T \underline{q} = \underline{a}^T(\underline{b} - \lambda \underline{a}) = 0$

$$\underline{a}^T \underline{b} - \lambda \underline{a}^T \underline{a} = 0$$



Using trig

$$\frac{p}{b} = \cos\theta$$

$$p = b\cos\theta$$

$$\underline{p} = p\,\frac{\underline{a}}{a} = \underline{a}\,\frac{b\cos\theta}{a}$$

$$= \underbrace{\left(\frac{b\cos\theta}{a}\right)}_{\lambda}\underline{a}$$

$\boxed{4}$

$$\lambda(\underline{a}^T \underline{a}) = \underline{a}^T \underline{b}$$

$$\lambda = \frac{\underline{a}^T \underline{b}}{\underline{a}^T \underline{a}}$$

$$\underline{p} = \underline{a}\lambda = \underline{a}\left(\frac{\underline{a}^T \underline{b}}{\underline{a}^T \underline{a}}\right) = \frac{1}{\underline{a}^T \underline{a}}\left((\underline{a}\,\underline{a}^T)\underline{b}\right)$$

$$= \frac{1}{\underline{a}^T \underline{a}}\left(\underbrace{\underline{a}\;\underline{a}^T}_{\text{square matrix}}\;\underline{b}\right)$$

$$\underline{p} = \underbrace{\left(\frac{\underline{a}\,\underline{a}^T}{\underline{a}^T \underline{a}}\right)}_{P}\underline{b}$$

P is called the projection matrix.

$\boxed{5}$

Properties of P

① $P^2 = P$

$$P^2 = \frac{aa^T}{a^Ta}\,\frac{aa^T}{a^Ta} = \frac{1}{(a^Ta)^2}\left[a(a^Ta)a^T\right]$$

$$= \frac{\cancel{a^Ta}}{\cancel{(a^Ta)^2}}\,aa^T = \frac{aa^T}{a^Ta}$$

P is idempotent.

② P does not depend on $\|a\|$

μa

$$P = \frac{(\mu a)(\mu a^T)}{(\mu a)^T(\mu a)} = \frac{aa^T}{a^Ta}$$

$p = Pb$

$p = P(Pb) = P^2 b$

③ $P_1 = I - P$ is also a projection matrix

$\boxed{6}$

③
$$P_1^2 = (I-P)^2 = (I-P)(I-P)$$
$$= I - 2P + P^2$$
$$= I - 2P + P = I - P$$

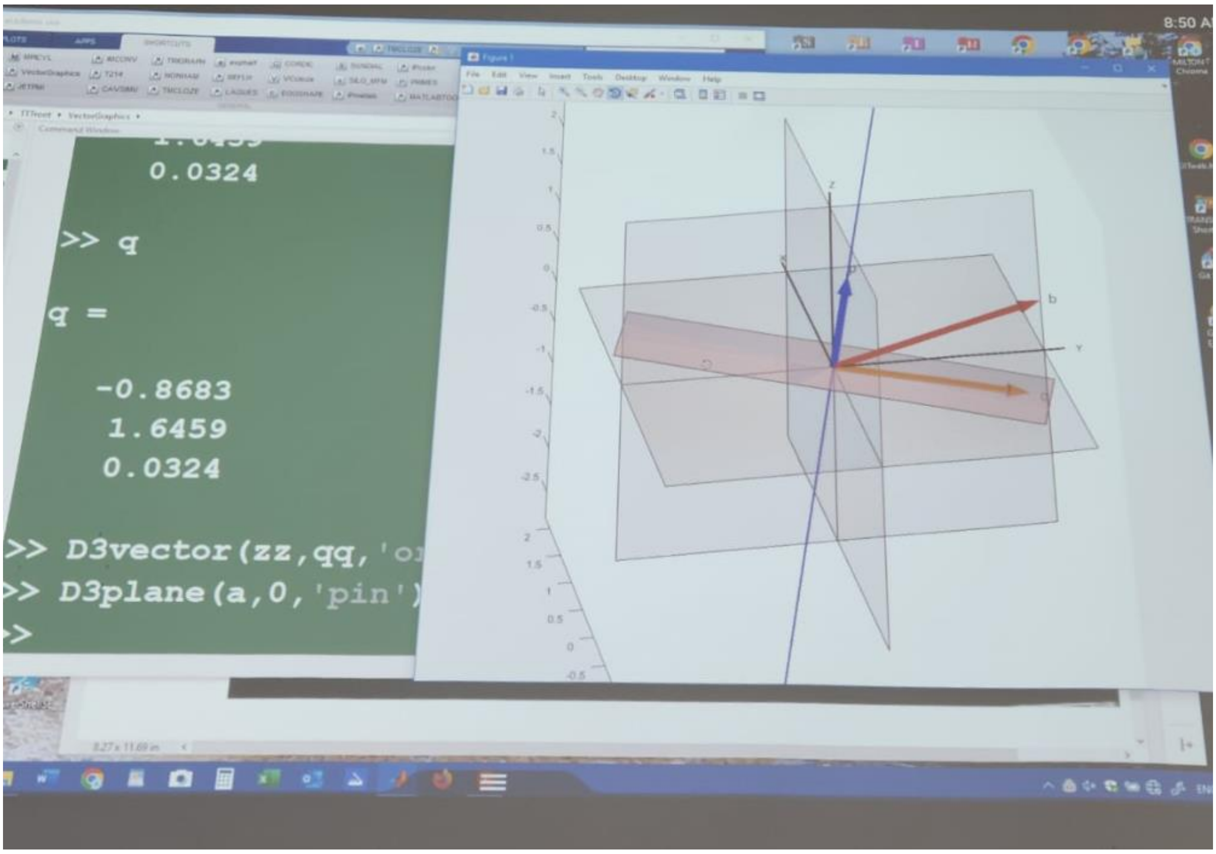
$q = b - p = b - Pb = (I-P)b$

P projects on line L

I − P " " plane orthogonal to line L.

$(P_1 + P)b = P_1 b + Pb = q + p = b$

Ib


0.0324
>> q
q =
-0.8683
1.6459
0.0324
>> D3vector(zz,qq,'o
>> D3plane(a,0,'pin')
>>