AM214-2023: LECTURE 9

Lecture 9 — Linear Spaces

1

Password for DIY-tests (Sunlearn): [redacted]

Password for Notes/Problems (Website): [redacted]

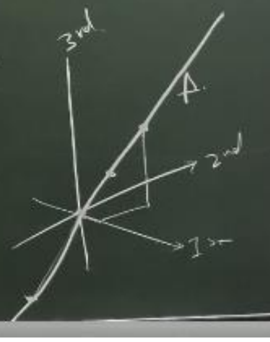
Set Notation:

such that

$$A = \left\{ \underline{x} \in \mathbb{R}^3 \;\middle|\; \underbrace{\underline{x} \in \lambda \begin{bmatrix}1\\2\\3\end{bmatrix},\ \lambda \in \mathbb{R}}_{\text{Constraints}} \right\}$$

Typical

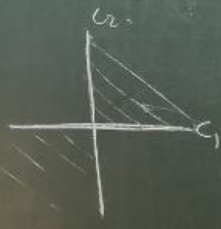
Host space

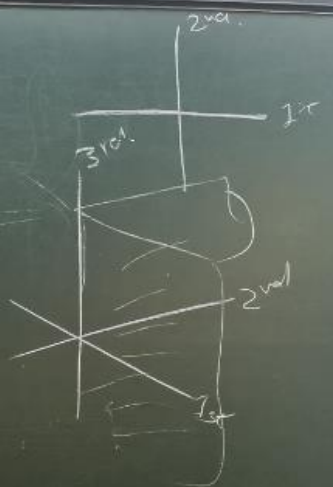


$$B = \left\{ \underline{b} \in \mathbb{R}^2 \;\middle|\; b_2 = 0 \right\}$$

2

$$C = \left\{ \underline{c} \in \mathbb{R}^3 \;\middle|\; c_1 c_2 \geq 0 \right\}$$





Vector space:

Maths: $+$, λ

AM214: only vectors.

VECTOR SPACE

$\boxed{3}$

Two closure rules: Let V be the VS.

Closed under addition: If $\underline{a}, \underline{b} \in V$, then $\underline{a}+\underline{b} \in V$.

Closed under sca. mult: $\underline{a} \in V$, $\lambda \in \mathbb{R}$, then $\lambda \underline{a} \in V$.

- $\underline{0}$ is always in a VS.
- VS is infinite

Examples:

$\boxed{4}$

$$A = \left\{ \underline{a} \in \mathbb{R}^3 \;\middle|\; \underline{a} = \lambda \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \lambda \in \mathbb{R} \right\}$$

$$B = \left\{ \underline{b} \in \mathbb{R}^2 \;\middle|\; b_2 = 0 \right\}$$

inequality

$$C = \left\{ \underline{c} \in \mathbb{R}^2 \;\middle|\; c_1 \geq 0 \right\}$$

$$D = \left\{ \underline{x} \in \mathbb{R}^2 \;\middle|\; x_1 x_2 \geq 0 \right\}$$

$$E = \left\{ \underline{x} \in \mathbb{R}^3 \;\middle|\; \underline{x} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R} \right\}$$

$\boxed{5}$

A: $\underline{a} = \lambda \begin{bmatrix}1\\2\\5\end{bmatrix}$, $\underline{b} = \mu \begin{bmatrix}1\\2\\5\end{bmatrix}$

$\underline{a}+\underline{b} = (\lambda+\mu)\begin{bmatrix}1\\2\\5\end{bmatrix}$ Rule 1 ✓

$\kappa \underline{a} = \kappa\left(\lambda \begin{bmatrix}1\\2\\5\end{bmatrix}\right) = \kappa\lambda \begin{bmatrix}1\\2\\5\end{bmatrix}$ Rule 2 ✓

Is a VS.

B: $\underline{b}_1 = \begin{bmatrix}x\\0\end{bmatrix}$, $\underline{b}_2 = \begin{bmatrix}y\\0\end{bmatrix}$ Is a VS.

C: $\underline{c}_1 = \begin{bmatrix}3\\-2\end{bmatrix}$, $\underline{c}_2 = \begin{bmatrix}10\\2\end{bmatrix}$ Rule 1 ✓ Rule 2 ✗ || Not a VS.

$\boxed{6}$

$D = \{ \underline{x} \in \mathbb{R}^2 \mid x_1 x_2 = 0 \}$

$\underline{d}_1 = \begin{bmatrix}5\\0\end{bmatrix}$, $\underline{d}_2 = \begin{bmatrix}0\\-2\end{bmatrix}$ $\underline{d}_1+\underline{d}_2 = \begin{bmatrix}5\\-2\end{bmatrix}$ Rule 1 ✗ Rule 2 ✓

Not a VS.

E: $\underline{e}_1 = \alpha \begin{bmatrix}1\\1\\0\end{bmatrix} + \beta \begin{bmatrix}0\\1\\1\end{bmatrix}$

$\underline{e}_2 = \gamma \begin{bmatrix}1\\1\\0\end{bmatrix} + \delta \begin{bmatrix}0\\1\\1\end{bmatrix}$

$\underline{e}_1 + \underline{e}_2 = (\alpha+\gamma)\begin{bmatrix}1\\1\\0\end{bmatrix} + (\beta+\delta)\begin{bmatrix}0\\1\\1\end{bmatrix}$

$\boxed{7}$

$$E' = \left\{ \underline{b} \in \mathbb{R}^3 \;\middle|\; \underline{b} = A\underline{x},\ A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix},\ \underline{x} \in \mathbb{R}^2 \right\}$$

$$\underline{b} = A\underline{x} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} x_2$$

$$E'' = \left\{ \underline{b} \in \mathbb{R}^3 \;\middle|\; \underline{b} = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} -10 \\ -10 \\ 0 \end{bmatrix} + \delta \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix},\ \alpha, \beta, \gamma, \delta \in \mathbb{R} \right\}$$

Geometrical view of VS in $\mathbb{R}^3$:

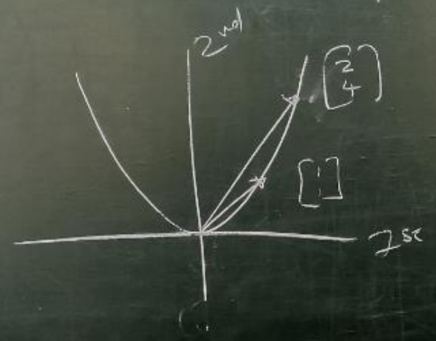
$A_0 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ Point (Origin)

$A_1 = \left\{ \underline{b} \in \mathbb{R}^3 \;\middle|\; \underline{b} = \lambda \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix},\ \lambda \in \mathbb{R} \right\}$ Line goes through origin.

$\boxed{8}$

$A_2 = \left\{ \underline{b} \in \mathbb{R}^3 \;\middle|\; \underline{b} = \lambda \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 6 \\ 9 \end{bmatrix},\ \lambda, \mu \in \mathbb{R} \right\}$ Plane through origin.

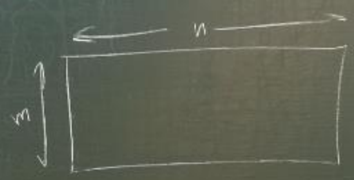
$A_3 = \mathbb{R}^3$ 3D-space.



$$F = \left\{ \underline{x} \in \mathbb{R}^2 \;\middle|\; x_2 = x_1^2 \right\}$$

COLUMN SPACES

$A \in \mathbb{R}^{m \times n}$



$$\mathrm{Col}(A) = \{ \underline{b} \in \mathbb{R}^m \mid \underline{b} = A\underline{x},\ \underline{x} \in \mathbb{R}^n \}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$