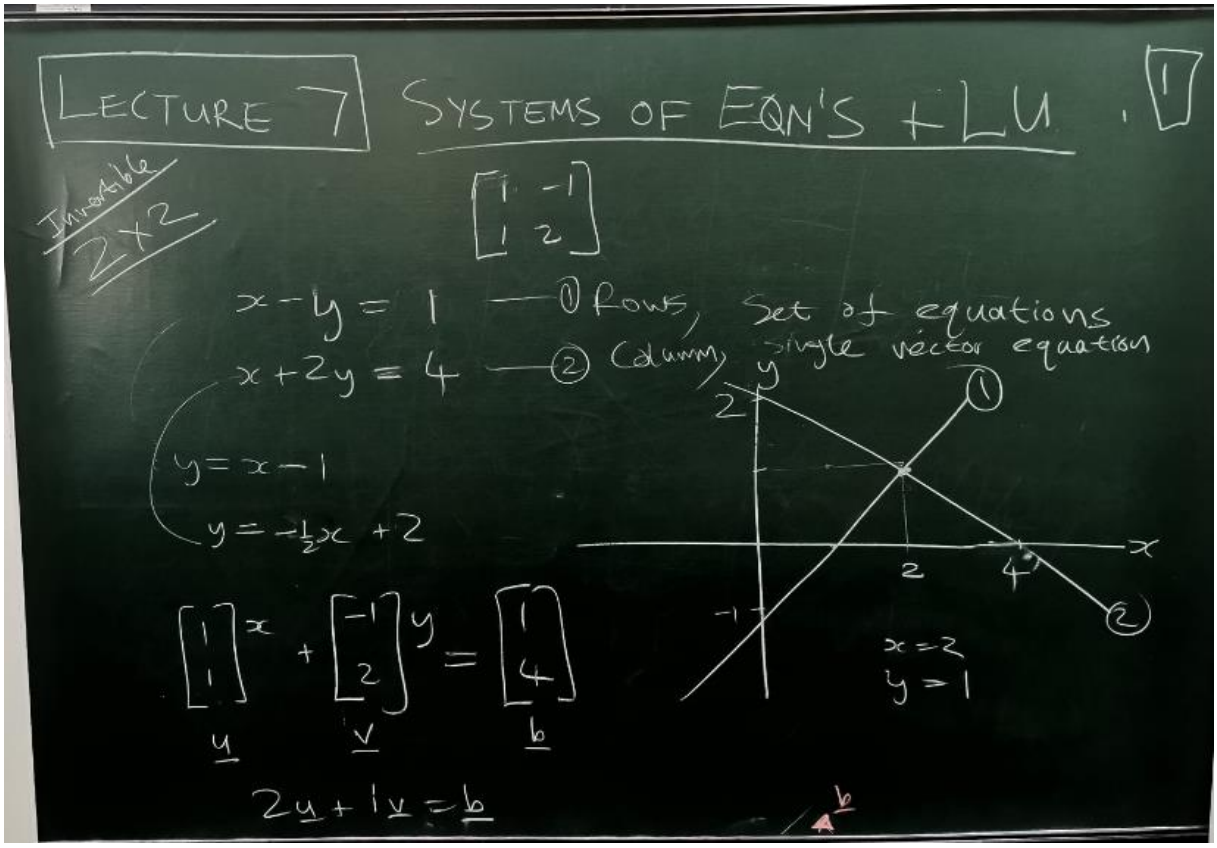
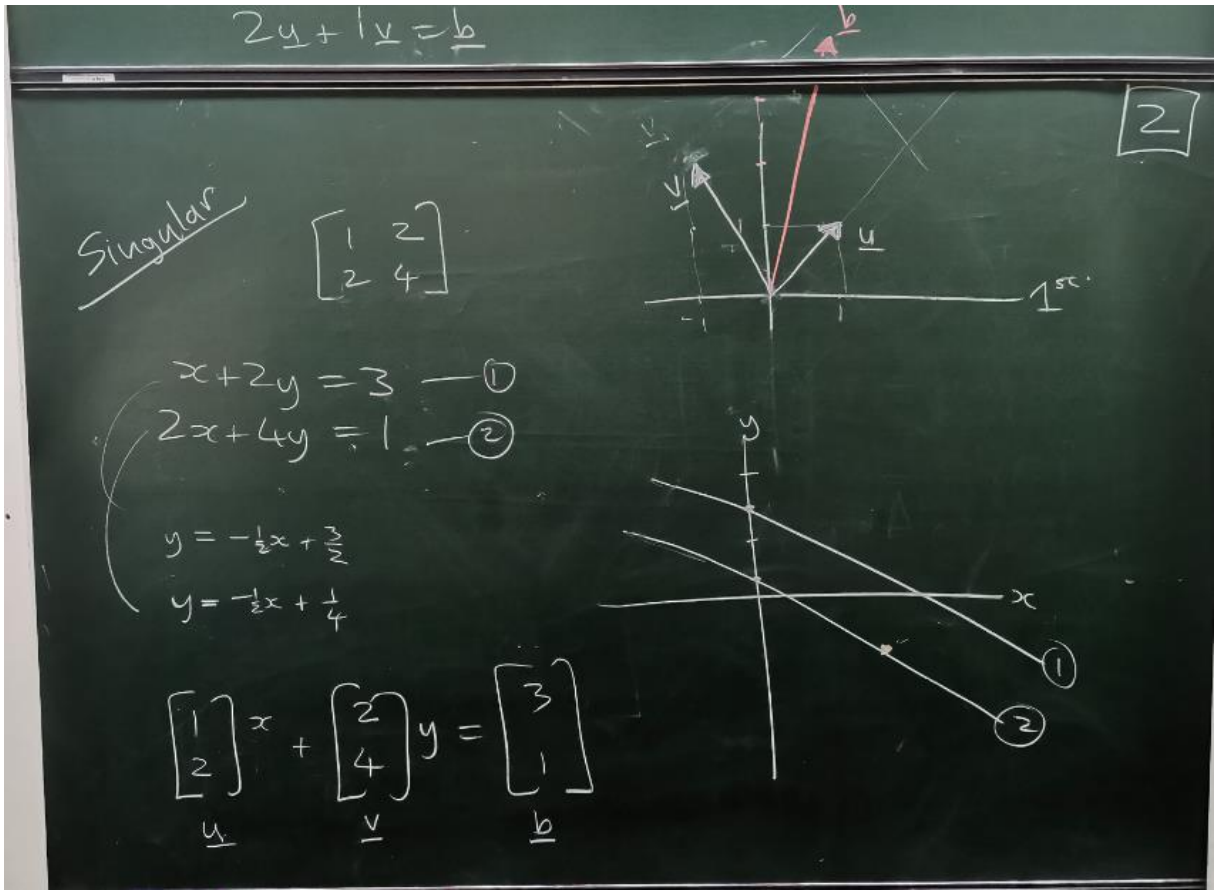
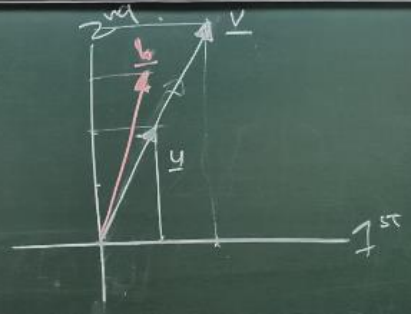
AM214-2023: LECTURE 7





3



$A \in \mathbb{R}^{3\times 3}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & -5 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 5.5 \end{bmatrix}$$

Row:

$$x + 2y + 3z = 7 \quad (1)$$
$$4x + 2y - 5z = 2 \quad (2)$$
$$3x + y + z = \frac{11}{2} \quad (3)$$

4

Column:

$$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} y + \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} z = \begin{bmatrix} 7 \\ 2 \\ 5.5 \end{bmatrix}$$

$\underline{u}$ $\underline{v}$ $\underline{w}$ $\underline{b}$

Singular

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 0 & 1 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} 7 \\ 2 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -2 & -1 \end{bmatrix} \qquad \underline{c} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

LU-decomp. Ekempel:

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$$A = \begin{bmatrix} 2 & 1 & 5 \\ 8 & 2 & 19 \\ -6 & -13 & -16 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} -1 \\ -7 \\ -16 \end{bmatrix}$$

LU-decomp

$$\begin{bmatrix} 2 & 1 & 5 \\ 0 & -2 & -1 \\ 0 & -10 & -1 \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 1 & 5 \\ 0 & -2 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$l_{21} = \frac{8}{2} = 4$$

$$l_{31} = \frac{-6}{2} = -3$$

$$l_{32} = \frac{-10}{-2} = 5$$

$LU = A$

① LU

② Forw. sylose $L\underline{c} = \underline{b}$

③ Backw. sylos. $U\underline{x} = \underline{c}$

$A\underline{x} = \underline{b}$

$L(U\underline{x}) = \underline{b}$

$L\underline{c} = \underline{b}$ ← Forw. sylose.

$U\underline{x} = \underline{c}$ ← Backw.

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$L\underline{c} = \underline{b}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ -16 \end{bmatrix}$$

$c_1 = -1$

$4(-1) + c_2 = -7$

$c_2 = -3$

$(-3)(-1) + 5(-3) + c_3 = -16$

$c_3 = -4$

$$\underline{c} = \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$$

$U\underline{x} = \underline{c}$

$$\begin{bmatrix} 2 & 1 & 5 \\ 0 & -2 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$$

$4z = -4, \quad z = -1$

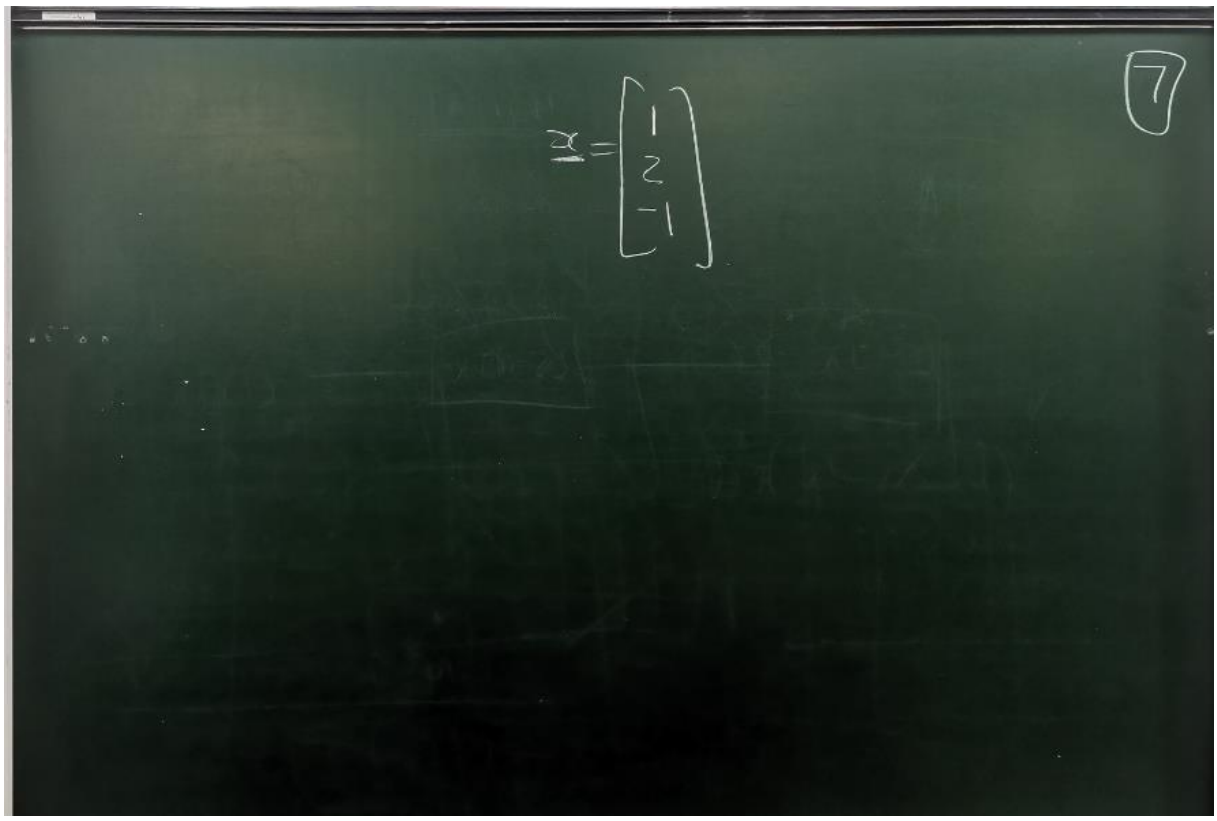
$-2y - 1(-1) = -3$

$-2y = -4$

$y = 2$

$2x + 1(2) + 5(-1) = -1$

$x = 1$


7
x =
1
2
-1