

AM214-2023: LECTURE 7

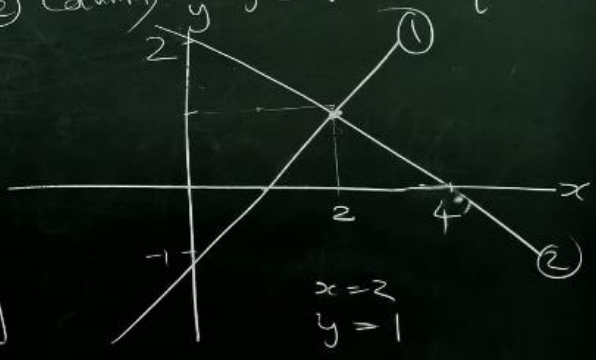
LECTURE 7 SYSTEMS OF EQN'S + LU [1]

Invertible
2x2

$$\begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$$

$x - y = 1$ — ① Rows, Set of equations
 $x + 2y = 4$ — ② Column, single vector equation

$y = x - 1$
 $y = -\frac{1}{2}x + 2$



$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 2 \end{bmatrix} y = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$2y + 1x = 4$$

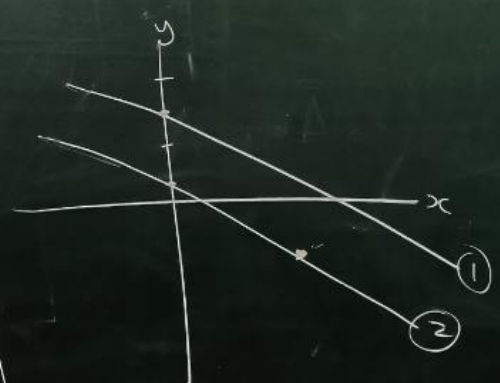
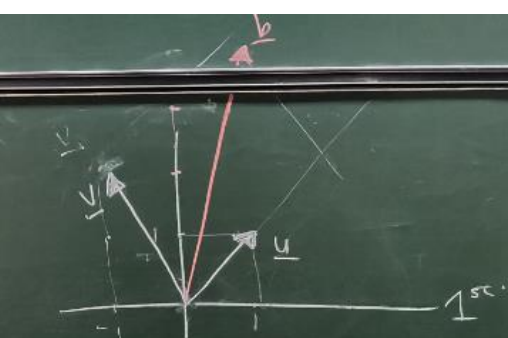
$$2y + 1x = 4$$

Singular

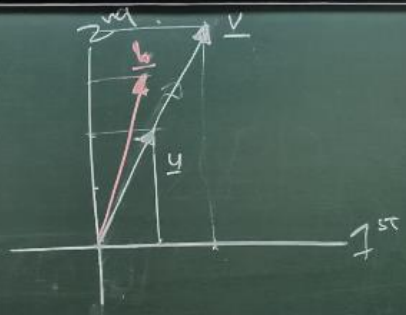
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$x + 2y = 3$ — ①
 $2x + 4y = 1$ — ②

$y = -\frac{1}{2}x + \frac{3}{2}$
 $y = -\frac{1}{2}x + \frac{1}{4}$



$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 4 \end{bmatrix} y = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



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$A \in \mathbb{R}^{3 \times 3}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & -5 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 5.5 \end{bmatrix}$$

- Row:
- $x + 2y + 3z = 7$ — ①
 - $4x + 2y - 5z = 2$ — ②
 - $3x + y + z = \frac{11}{2}$ — ③

Column:

$$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} y + \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} z = \begin{bmatrix} 7 \\ 2 \\ 5.5 \end{bmatrix}$$

$\underline{u} \quad \underline{v} \quad \underline{w} \quad \underline{b}$

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Singular

$$B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 0 & 1 \end{bmatrix}$$

$$\underline{b} = \begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix}$$



$$C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\underline{c} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

LU-decomp. Example

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$$A = \begin{bmatrix} 2 & 1 & 5 \\ 8 & 2 & 19 \\ -6 & -13 & -16 \end{bmatrix}$$

$$b = \begin{bmatrix} -1 \\ -7 \\ -16 \end{bmatrix}$$

- ① LU
- ② Forw. subst. $Lc = b$
- ③ Backw. subst. $Ux = c$

LU-decomp

$$\begin{bmatrix} 2 & 1 & 5 \\ 0 & -2 & -1 \\ 0 & -10 & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 1 & 5 \\ 0 & -2 & -1 \\ 0 & 0 & 4 \end{bmatrix}$$

$$l_{21} = \frac{8}{2} = 4$$

$$l_{31} = \frac{-6}{2} = -3$$

$$l_{32} = \frac{-10}{-2} = 5$$

$$Ax = b$$

$$L(Ux) = b$$

$$Lc = b \leftarrow \text{Forw. subst.}$$

$$Ux = c \leftarrow \text{Backw.}$$

$LU = A$

$Lc = b$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -7 \\ -16 \end{bmatrix}$$

$$c_1 = -1$$

$$4(-1) + c_2 = -7$$

$$c_2 = -3$$

$$c = \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$$

$$(-3)(-1) + 5(-3) + c_3 = -16$$

$$c_3 = -4$$

$Ux = c$

$$\begin{bmatrix} 2 & 1 & 5 \\ 0 & -2 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$$

$$2x + 1(2) + 5(-1) = -1$$

$$-4z = -4, z = 1$$

$$-2y - 1(1) = -3$$

$$-2y = -4$$

$$y = 2$$

$$x = 1$$

$$\underline{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

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