



AM214-2023: LECTURE 4

LECTURE 4 SQUARE MATRICES, EQUATIONS 1

TRANSPOSE:  $\underline{a}$ ,  $\underline{q}^T$

$A \in \mathbb{R}^{(m \times n)}$

$A^T =$  



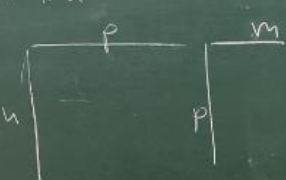
RULES:

- ①  $(A^T)^T = A$
- ②  $(A+B)^T = A^T + B^T$
- ③  $(AB)^T = B^T A^T$

$\underline{q}^T \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$   
 $\underline{b}^T \underline{a} = b_1 a_1 + b_2 a_2 + b_3 a_3$   
 $(\underline{q}^T \underline{b})^T = (a_1 b_1 + a_2 b_2 + a_3 b_3)^T$   
 $\underline{b}^T \underline{q}^T = \underline{b}^T \underline{q}$

$A B = \underline{AB}$

$m \times p \quad p \times n \quad m \times n$

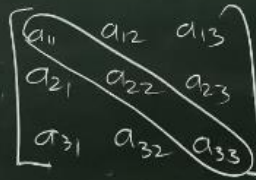
$(AB)^T =$    $= B^T A^T$

$(ABC)^T = [(AB)C]^T = C^T (AB)^T = C^T B^T A^T$

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SQUARE MAT'S  $n \times n$ :  $A \in \mathbb{R}^{n \times n}$

① diag

$A =$   diagonal

$(A \setminus (CD))$   
 $(ABC)D$

③ Trace

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$$\text{tr}(A) = a_{11} + a_{22} + a_{33}$$

④ Identity matrix (for multiplication)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AI = IA = A$$

⑤ Inverses

$$BA = I$$

B is a left-inverse of A

$$AC = I$$

C is a right-inverse of A.

$$B = BI = B(AC) = (BA)C = IC = C$$

$$AA^{-1} = I$$

Inverse of 2x2 matr.

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$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \frac{1}{ad-bc}$$

$$= \frac{1}{ad-bc} \begin{bmatrix} ad-bc & -ab+ab \\ cd & -bc+ad \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A^{-1}$  exists only if  $ad-bc \neq 0$

When  $\det(A) = 0$ , (other words  $A^{-1}$  does not exist) then A called "singular".

Transp + inverse

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$$(A^T)^{-1} \stackrel{?}{=} (A^{-1})^T$$

Proof:  $A$  is  $n \times n$ ,  $A^{-1}$  is  $n \times n$   
 $AA^{-1} = I$

Transpose:  $(AA^{-1})^T = I$

$$(A^{-1})^T A^T = I$$

Post multiply by  $(A^T)^{-1}$ :

$$(A^{-1})^T A^T (A^T)^{-1} = I (A^T)^{-1}$$
$$(A^{-1})^T I = (A^T)^{-1}$$
$$(A^{-1})^T = (A^T)^{-1}$$

MATRIX EQUATIONS

$X, A, B$  all  $n \times n$ .

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Make  $X$  the subject of formula:  
(whenever some inverse is required, it exists)

(a)  $XA + X = B$

$$X(A+I) = B$$

Postmult. by  $(A+I)^{-1}$

$$\cancel{X(A+I)}(A+I)^{-1} = B(A+I)^{-1}$$

$$X = B(A+I)^{-1}$$

~~$(a+b)^{-1} = a^{-1} + b^{-1}$~~   
 ~~$(a^2+b^2)^{-1} = a^{-2} + b^{-2}$~~

(b)

$$(b) \quad A = T X T^{-1}$$

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Pre-mult by  $T^{-1}$ :  $T^{-1}A = (T^{-1}T)X T^{-1}$   
 $= X T^{-1}$

Post-mult by  $T$   $T^{-1}AT = X$

$$(c) \quad A^{-1}(X - I)A + (X - A^{-1})B^T A = A - I + X B^T A$$
$$A^{-1}XA - \underbrace{A^{-1}IA}_I + XB^T A - A^{-1}B^T A = A - I + XB^T A$$