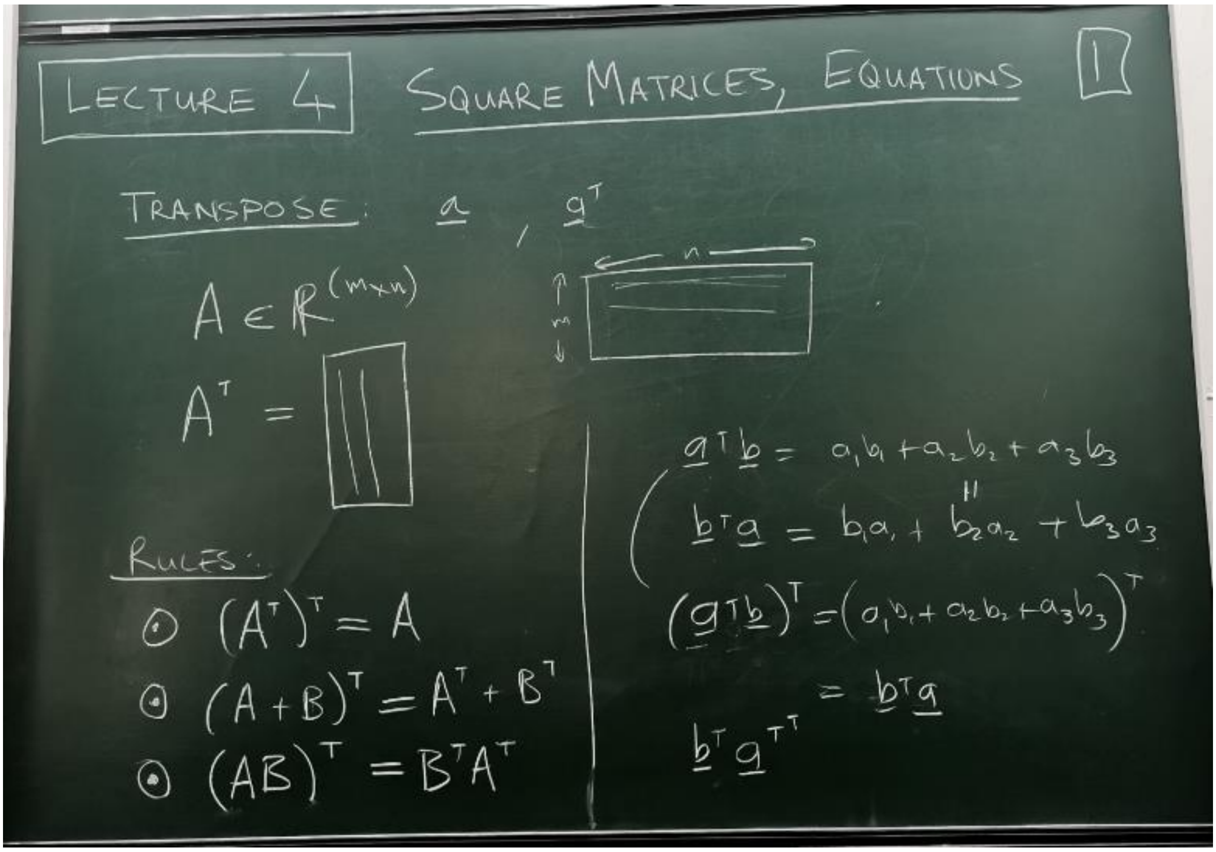
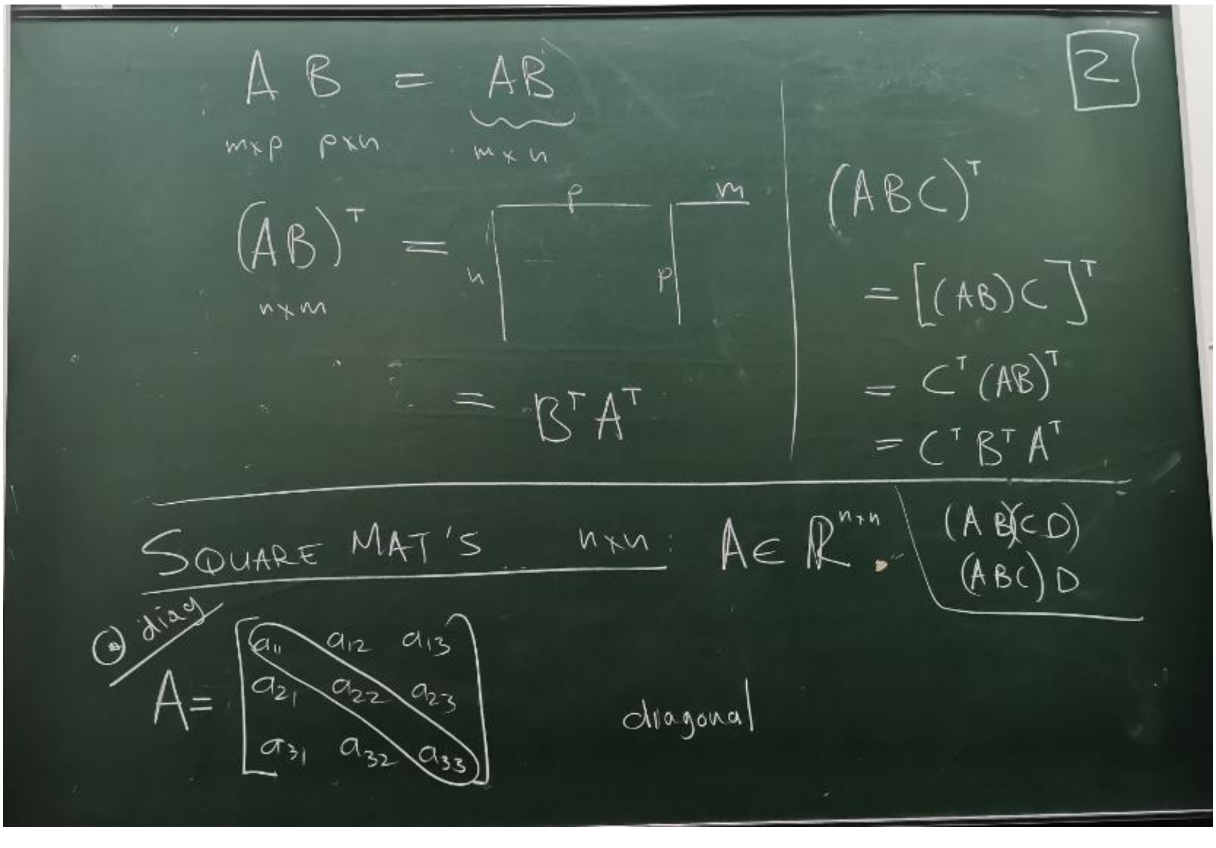
AM214-2023: LECTURE 4





$\boxed{3}$

Trace

$$tr(A) = a_{11} + a_{22} + a_{33}$$

Identity matrix (for multiplication)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad AI = IA = A$$

Inverses

$BA = I$ B is a left-inverse of A

$AC = I$ C is a right inverse of A.

$$B = BI = B(AC) = (BA)C = IC = C$$

$$AA^{-1} = I$$

$\boxed{4}$

Inverse of 2x2 matr.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\frac{1}{ad-bc}$$

$$= \frac{1}{ad-bc}\begin{bmatrix} ad-bc & \overset{0}{\cancel{-ab+ab}} \\ \overset{0}{\cancel{cd-cd}} & -bc+ad \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A^{-1} exists only if $ad-bc \neq 0$

When $\det(A)=0$, (other words A^{-1} does not exist) then A called "singular".

$\boxed{5}$

Transp + inverse

$$(A^T)^{-1} \overset{?}{\sim} (A^{-1})^T$$

Proof: A is $n \times n$, A^{-1} e...

$$AA^{-1} = I$$

Transpose: $(AA^{-1})^T = I$

$$(A^{-1})^T A^T = I$$

Post multiply by $(A^T)^{-1}$: $(A^{-1})^T \underbrace{A^T (A^T)^{-1}} = I (A^T)^{-1}$

$$(A^{-1})^T I = (A^T)^{-1}$$

$$(A^{-1})^T = (A^T)^{-1}$$

MATRIX EQUATIONS

X, A, B all $n \times n$. $\boxed{6}$

Make X the subject of formula:

(whenever some inverse is required, it exists)

(a) $XA + X = B$

$$X\underbrace{(A+I)} = B$$

Postmult. by $(A+I)^{-1}$

$$X\cancel{(A+I)(A+I)^{-1}} = B(A+I)^{-1}$$

$$X = B(A+I)^{-1}$$

(b)

$\cancel{(a+b)^{-1}}$ a^{-1} b^{-1}

$\cancel{\sqrt{a^2+b^2} = a+b}$

$\boxed{7}$

(b) $A = TXT^{-1}$

Premult by T^{-1}: $T^{-1}A = (T^{-1}T)XT^{-1}$

$= XT^{-1}$

Postmult by T

$T^{-1}AT = X$

(c) $A^{-1}(X - I)A + (X - A^{-1})B^TA = A - I + XB^TA$

$A^{-1}XA - \underset{I}{\cancel{A^{-1}IA}} + XB^TA - A^{-1}B^TA = A - I + XB^TA$