

AM214-2023: LECTURE 3

LECTURE 3 MATRICES, NOTATION, MULTIPLICATION 1

NOTATION: A is in $\mathbb{R}^{3 \times 4}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$$A = \begin{bmatrix} | & | & | & | \\ \underline{a}_1 & \underline{a}_2 & \underline{a}_3 & \underline{a}_4 \\ | & | & | & | \end{bmatrix}$$


$$B = \begin{bmatrix} - & \underline{b}_1^T & - \\ - & \underline{b}_2^T & - \\ - & \underline{b}_3^T & - \end{bmatrix}$$

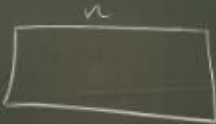
$\underline{a}_k \in \mathbb{R}^3, k=1, \dots, 4$

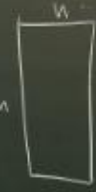
$A \in \mathbb{R}^{3 \times 4}, A$ is 3×4

Annotations: "which row" points to the first row of A; "which column" points to the first column of A; "first" points to the first row of A; "k-th" points to the k-th column of A.

TYPES: 2

$m = n$  square matrix

$m < n$  landscape matrix

$m > n$  portrait matrix

$$\begin{aligned} 3x + 2y + 5z &= 4 \\ -x + 7y + 4z &= 10 \end{aligned}$$

$$\begin{bmatrix} 3 & 2 & 5 \\ -1 & 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Annotations: "coef. matrix" points to the matrix; "right hand vector" points to the vector [4, 10]^T; "x = unknown" points to the vector [x, y, z]^T.

RULES:

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EQUALITY: $A = B$, $a_{ij} = b_{ij}$

ADD: $A + B = C$, $c_{ij} = a_{ij} + b_{ij}$ $\{1, 2, 3\} = \{3, 1, 2\}$

SCALAR MULT: $B = \lambda A$, $b_{ij} = \lambda a_{ij}$ $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \neq \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \end{bmatrix}$

MULTIPLICATION: ... later.

NULL MATRIX: $\mathbf{O} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

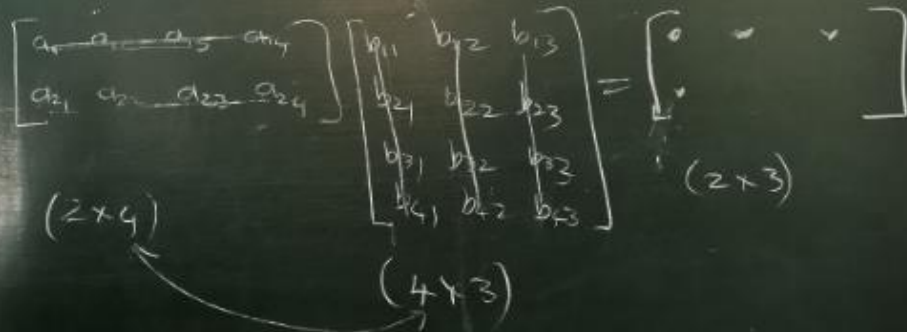
MATRIX MULT.

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ab AB
dot prod.

⊙ AB is defined only if

"inner sizes are the same"



$\sum^i y = \text{dot prod.}$

⊙ Commutativity (NO!)

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$AB, BA \rightarrow$ same sizes only for $n \times n$
 $AB \neq BA$

⊙ $AB = O$

$$\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LAWS:

MATR. ADD.

$$A+B = B+A \quad \text{Comm.} \checkmark$$

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$$A+(B+C) = (A+B)+C \quad \text{Associative} \checkmark$$

SCA. MULT.

$$\lambda A = A \lambda \quad \text{Comm.} \checkmark$$

MATR. MULT:

⊙ Not commutative \times

$$\odot A(BC) = (AB)C \quad \text{Assoc.} \checkmark$$

$$\odot A(B+C) = AB+AC$$

Left-distributivity \checkmark

$$\odot (A+B)C = AC+BC \quad \checkmark$$

TYPICAL PRODUCTS

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⊙ $x^T y = \text{---} = \text{---}$

⊙ $AB = \begin{matrix} \text{---} \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ \text{---} \end{matrix} = \begin{matrix} \text{---} \\ \text{---} \end{matrix}$

⊙ $Ax = \begin{matrix} \text{---} \\ \text{---} \end{matrix} \begin{matrix} \text{---} \\ \text{---} \end{matrix} = \begin{matrix} \text{---} \\ \text{---} \end{matrix}$

⊙ $y^T A = \text{---} \begin{matrix} \text{---} \\ \text{---} \end{matrix} = \text{---}$

⊙ $x y^T = \text{---} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$

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MATRIX EQUATIONS

$$C = A + B$$

Pre-Mult by S:

$$SC = SA + SB$$

Post multiply by S:

$$CS = AS + BS$$

Four views of matrix mult

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Rows \times Cols = Dots

$$\textcircled{1} \begin{bmatrix} \equiv \\ \equiv \\ \equiv \end{bmatrix} \begin{bmatrix} | \\ | \\ | \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

Mat \times Cols = Cols

$$\textcircled{2} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \begin{bmatrix} | \\ | \\ | \end{bmatrix} = \begin{bmatrix} | \\ | \\ | \end{bmatrix}$$

Rows \times Mat = Rows

$$\textcircled{3} \begin{bmatrix} \equiv \\ \equiv \\ \equiv \end{bmatrix} \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} \equiv \\ \equiv \\ \equiv \end{bmatrix}$$

Cols \times Row =

$$\textcircled{4} \begin{bmatrix} | \\ | \\ | \end{bmatrix} \begin{bmatrix} \equiv \\ \equiv \\ \equiv \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} ax+bz & ay+bw \\ cx+dz & cy+dw \end{bmatrix}$$

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$$= \begin{bmatrix} ax & ay \\ cx & cy \end{bmatrix} + \begin{bmatrix} bz & bw \\ dz & dw \end{bmatrix}$$

$$= \begin{bmatrix} a \\ c \end{bmatrix} \begin{bmatrix} x & y \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} \begin{bmatrix} z & w \end{bmatrix}$$