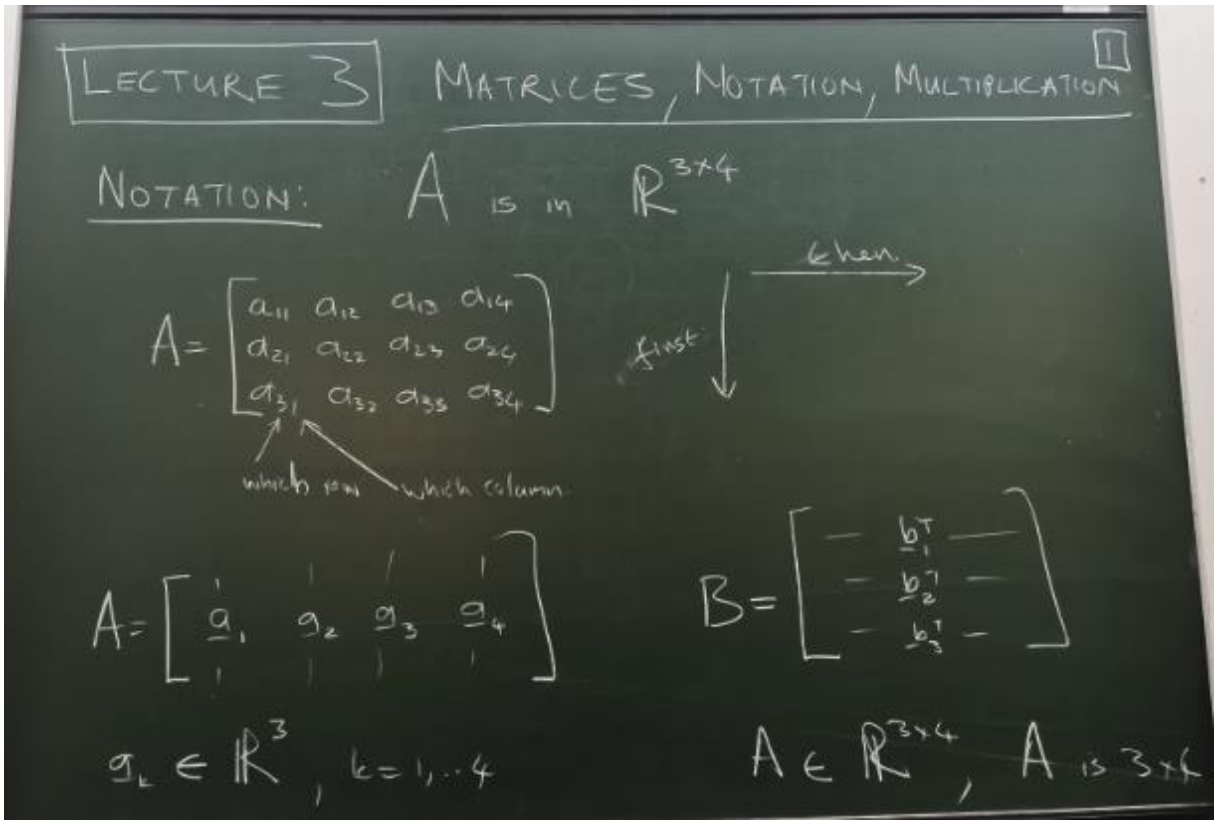
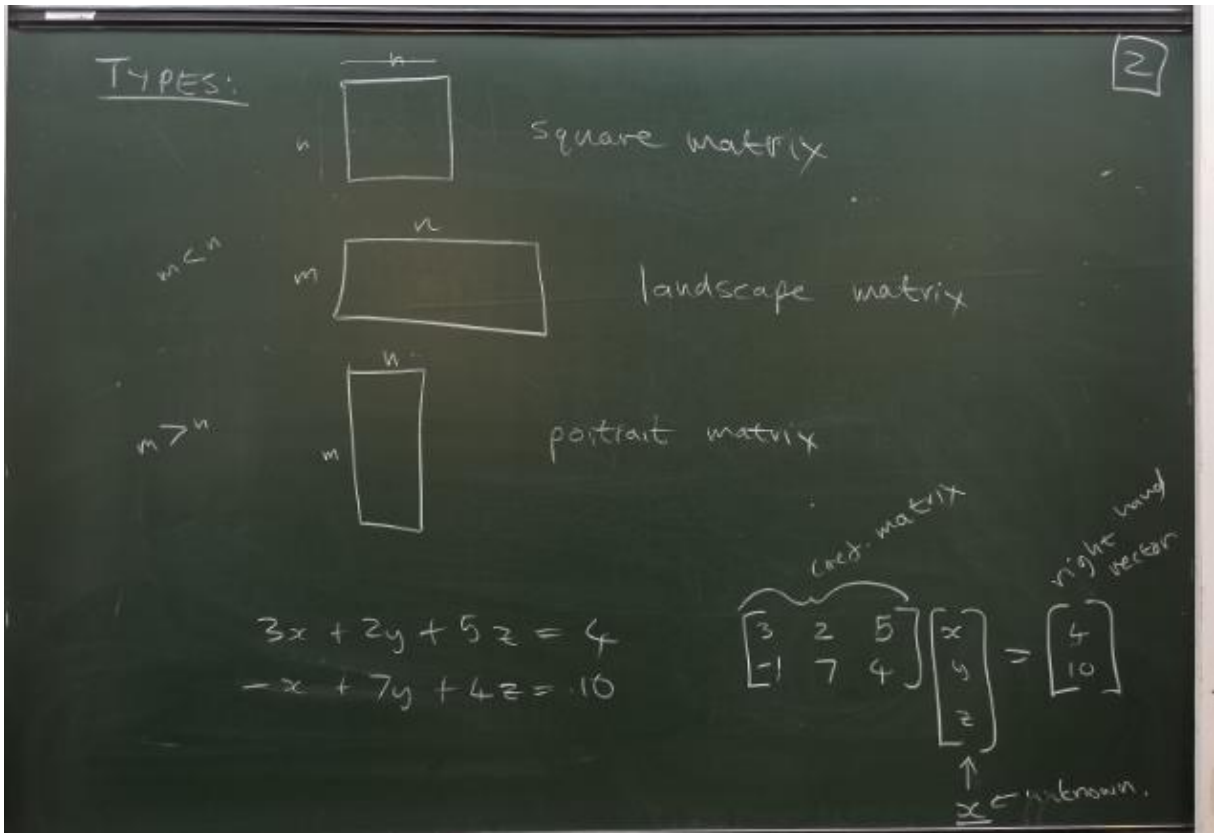
AM214-2023: LECTURE 3





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RULES:

EQUALITY: $A = B$, $a_{ij} = b_{ij}$

ADD: $A + B = C$, $c_{ij} = a_{ij} + b_{ij}$ $\quad \{1, 3, 3\} = \{3, 1, 2\}$

SCALAR MULT: $B = \lambda A$, $\quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \neq \begin{bmatrix} 1 & 3 & 2 \\ 4 & 5 & 6 \end{bmatrix}$

$b_{ij} = \lambda a_{ij}$

MULTIPLICATION: .... later.

NULL MATRIX: $O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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MATRIX MULT.

ab AB defined.

① AB is defined only if.

"inner sizes are the same"

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

(2×4) $\quad (4 \times 3)$ $\quad (2 \times 3)$

$x^T y$ = dot prod.

5

- Commutativity. (NO!)

  AB, BA → same sizes only for n×n

  $AB \neq BA$

- $AB = O$

$$\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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LAWS:

MATR. ADD.

$A+B = B+A$ Comm. ✓

$A+(B+C) = (A+B)+C$ Associative ✓

SCA. MULT.

$\lambda A = A\lambda$ Comm. ✓

MATR. MULT:

- Not commutative ✗
- $A(BC) = (AB)C$ Assoc. ✓
- $A(B+C) = AB+AC$ Left-distributivity ✓
- $(A+B)C = AC+BC$ ✓

TYPICAL PRODUCTS:

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① $x^T y$ =

② AB = (square)

③ Ax =

④ $y^T A$ =

8

① $x y^T$ =

rank-one matrix

MATRIX EQUATIONS

$C = A + B$

Pre-Multiply by S : $SC = SA + SB$

Post multiply by S : $CS = AS + BS$

Four views of matrix mult

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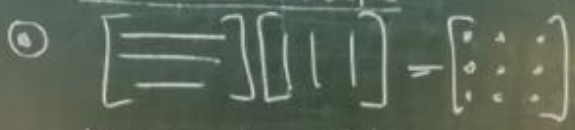
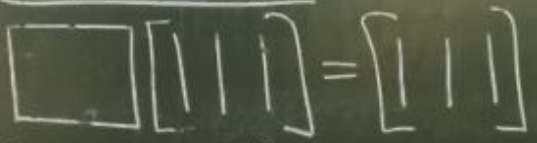
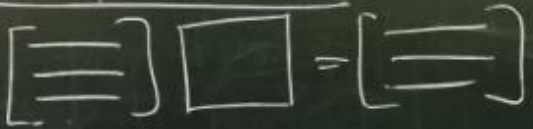
Rows x Cols = Dotp's



Mat x Cols = Cols



Rows x Mat = Rows



Cols x Row =

10

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} ax+bz & ay+bw \\ cx+dz & cy+dw \end{bmatrix}$$

$$= \begin{bmatrix} ax & ay \\ cx & cy \end{bmatrix} + \begin{bmatrix} bz & bw \\ dz & dw \end{bmatrix}$$

$$= \begin{bmatrix} a \\ c \end{bmatrix}\begin{bmatrix} x & y \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix}\begin{bmatrix} z & w \end{bmatrix}$$