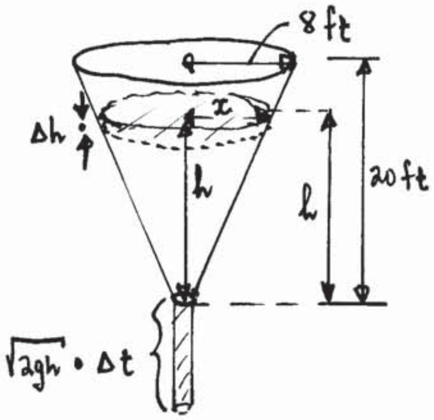
VRAAG 1 / QUESTION 1

Ex 2.8; Nr 15(a) and Ex 1.3; Nr 14



In the time period Δt, the water level decreases with a height of Δh, and the volume water decreases with ΔV.

From the sketch:

$$\Delta V = \pi x^2 \Delta h$$

and from proportionality we approximately have that: $x/h = 8/20$ ← simplification!

$$\Rightarrow x = \frac{2}{5} h$$

Consequently:

$$\Delta V = \pi \frac{4}{25} h^2 \Delta h$$

Furthermore, according to Toricelli's law, we have that:

$$\Delta V = -c\sqrt{2gh}\,(\Delta t)(A_h)$$

area of hole

$$\Rightarrow \pi \frac{4}{25} h^2 \Delta h = -c\sqrt{2gh}\,(\Delta t)(\pi r^2)$$

friction factor; radius of hole

Given: $c = 0{,}6$; $r = \frac{2}{12} = \frac{1}{6}$ ft ; $g = 32$ ft/s^2

$$\Rightarrow \frac{4}{25} h^2 \Delta h = -0{,}6\sqrt{64h}\,(\Delta t)\left(\frac{1}{6^2}\right)$$

$$\Rightarrow h^2 \Delta h = -\frac{5}{6}\sqrt{h}\,\Delta t$$

$$\Rightarrow \frac{\Delta h}{\Delta t} = -\frac{5}{6}\frac{1}{h^{3/2}}$$

Diff. eqn!

When $\Delta t \to 0 \Rightarrow \frac{dh}{dt} = -\frac{5}{6}\frac{1}{h^{3/2}}$; $h(0) = 20$

Separation of variables:

$$\int h^{3/2}\,dh = -\frac{5}{6}\int dt + C$$

$$\frac{2}{5} h^{5/2} = -\frac{5}{6} t + C$$

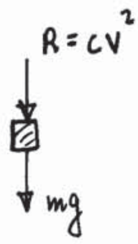
When $t = 0$, then $h = 20$: $\frac{2}{5} 20^{5/2} = C$

$$\Rightarrow \frac{2}{5} h^{5/2} = -\frac{5}{6} t + \frac{2}{5} 20^{5/2}$$

Tank is empty when $h = 0$, that is:

$$t = \left(\frac{6}{5}\right)\left(\frac{2}{5}\right) 20^{5/2} = 859 \text{ seconds}$$

VRAAG 2 / QUESTION 2



$$\oplus\uparrow \sum F_y = m(a_G)_y$$

$$-mg - cv^2 = m\frac{dv}{dt}$$

$$\frac{dv}{dt} = -g - \frac{c}{m}v^2$$
$$= -\frac{c}{m}\left(v^2 + \frac{mg}{c}\right)$$
$$= -\frac{c}{m}\left(v^2 + a^2\right); \text{ where } a^2 = \frac{mg}{c}$$

$$\Rightarrow \int \frac{1}{a^2+v^2}\,dv = -\frac{c}{m}\,dt + K$$

$$\Rightarrow \frac{1}{a}\arctan\left(\frac{v}{a}\right) = -\frac{c}{m}t + K$$

But $v(0) = V_0 \Rightarrow K = \frac{1}{a}\arctan\left(\frac{V_0}{a}\right)$

$$\Rightarrow \frac{1}{a}\arctan\left(\frac{V}{a}\right) = -\frac{c}{m}t + \frac{1}{a}\arctan\left(\frac{V_0}{a}\right)$$

Maximum height is where $V = 0$

$$\Rightarrow t_{max} = \frac{m}{ac}\arctan\left(\frac{V_0}{a}\right)$$

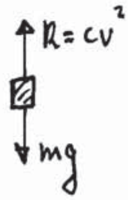
$$a = \sqrt{\frac{mg}{c}} = \sqrt{\frac{10(9,8)}{10^{-4}}} = 989,9495$$

$$\Rightarrow V_0/a = 200/989,9495 = 0,20203$$

$$\Rightarrow \frac{m}{ac} = \frac{10}{(989,9495)(10^{-4})} = 101,0153$$

$$\Rightarrow t_{max} = 101,0153\arctan(0,20203)$$
$$= 20,137 \text{ seconds}$$

When moving downwards:

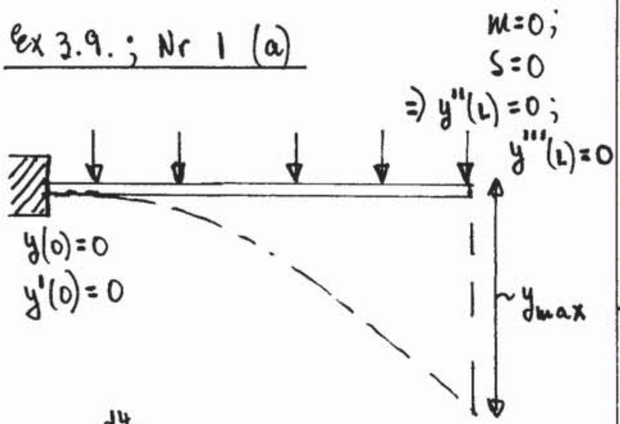


$$\oplus\downarrow \sum F_y = m(a_G)_y$$

$$mg - cV^2 = m\frac{dv}{dt}$$

QUESTION 3

Ex 3.9.; Nr 1 (a)



$$EI\frac{d^4y}{dx^4} = w(x) = w_0$$

$$\Rightarrow \frac{d^4y}{dx^4} = \frac{w_0}{EI} = A$$ ← given

$$y'''(x) = Ax + B$$

$$y'''(L) = AL + B = 0$$

$$\Rightarrow B = -AL$$

$$y''(x) = \tfrac{1}{2}Ax^2 + Bx + C$$

$$y''(L) = \tfrac{1}{2}AL^2 + BL + C = 0$$

$$\Rightarrow C = -\tfrac{1}{2}AL^2 - BL$$

$$= -\tfrac{1}{2}AL^2 - (-AL)L$$

$$= -\tfrac{1}{2}AL^2 + AL^2$$

$$= \tfrac{1}{2}AL^2$$

$$y'(x) = \tfrac{1}{6}Ax^3 + \tfrac{1}{2}Bx^2 + Cx + D$$

$$y'(0) = D = 0$$

$$y(x) = \tfrac{1}{24}Ax^4 + \tfrac{1}{6}Bx^3 + \tfrac{1}{2}Cx^2 + E$$

$$y(0) = E = 0$$

$$\Rightarrow y(x) = \tfrac{1}{24}Ax^4 + \tfrac{1}{6}(-AL)x^3 + \tfrac{1}{2}\left(\tfrac{1}{2}AL^2\right)x^2$$

$$= \tfrac{1}{24}Ax^4 - \tfrac{1}{6}ALx^3 + \tfrac{1}{4}AL^2x^2$$

$$= \frac{Ax^2}{24}\left(x^2 - 4Lx + 6L^2\right)$$

$$= \frac{w_0x^2}{24EI}\left(x^2 - 4Lx + 6L^2\right)$$

Ex 3.9; Nr 6 (a)

$$y_{max} = y(L) = \frac{w_0}{24EI}L^2\left(L^2 - 4L^2 + 6L^2\right)$$

$$= \frac{w_0}{24EI}\left(3L^4\right)$$

$$= \frac{w_0L^4}{8EI}$$

Ex 3.9; Nr 6 (b)

When the length of the beam is $L/2$, the solution becomes:

$$y(x) = \frac{w_0}{24EI}\cdot x^2\cdot\left(x^2 - 4\left(\frac{L}{2}\right)x + 6\left(\frac{L}{2}\right)^2\right)$$

$$= \frac{w_0}{24EI}\cdot x^2\cdot\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$$

$$y_{max\,[length = L/2]} = \frac{w_0}{24EI}\left(\frac{L}{2}\right)^2\left[\left(\frac{L}{2}\right)^2 - 2L\left(\frac{L}{2}\right) + \frac{3}{2}L^2\right]$$

$$= \frac{w_0}{24EI}\left(\frac{L^2}{4}\right)\left[\frac{L^2}{4} - L^2 + \frac{3}{2}L^2\right]$$

$$= \frac{w_0}{24EI}\left(\frac{L^2}{4}\right)\left(\frac{3L^2}{4}\right)$$

$$= \frac{w_0L^4}{128EI}$$

$$= \frac{1}{16}\cdot y_{max\,[length = L]}$$