TWB252 | Tut 2 | 2 Augustus / August 2017 | MEMO

VRAAG 1 / QUESTION 1

$R \propto v \Rightarrow R = kv$

$\oplus\uparrow \Sigma F_y = m(a_G)_y$

$-kv - mg = m\dfrac{dv}{dt}$

$\Rightarrow \dfrac{dv}{dt} = -\rho v - g$

with $\rho = k/m$.

Solution: $v = \left(v_0 + \dfrac{g}{\rho}\right)e^{-\rho t} - \dfrac{g}{\rho}$

where V_0 is the initial speed.

Given: $V_0 = 49$ and $\rho = 0{,}04$

$$\Rightarrow v = \left(49 + \frac{9{,}81}{0{,}04}\right)e^{-0{,}04t} - \frac{9{,}81}{0{,}04}$$

$$= 294{,}25\, e^{-0{,}04t} - 245{,}25$$

$$= \frac{dy}{dt}$$

$$\Rightarrow y = -7356{,}25\, e^{-0{,}04t} - 245{,}25t + C$$

When $t=0$, then $y=0$:

$$0 = -7356{,}25 + C$$

$$\Rightarrow C = 7356{,}25$$

$$\Rightarrow y = -7356{,}25\, e^{-0{,}04t} - 245{,}25t + 7356{,}25$$

The object attains its maximum height when $v=0$

$$\Rightarrow 294{,}25\, e^{-0{,}04t} - 245{,}25 = 0$$

$\Rightarrow t = 4{,}5538$ seconds

$\Rightarrow$ Maximum height:

$y(4{,}5538) = 108{,}1827$ meters

In the absence of air resistance, the maximum height is $122{,}5$ meters

VRAAG 2 / QUESTION 2

The logistic equation and its solution are as follows:

$$\frac{dP}{dt} = P(a - bP)\ ;\ P(0) = P_0$$

$$\Rightarrow P = \frac{\frac{a}{b}P_0}{P_0 + \left(\frac{a}{b} - P_0\right)e^{-at}} \quad \cdots \ (*)$$

When $t \to \infty$, then $P \to \frac{a}{b} = 10\ 000$;

Let $t = 0$ in 1990 $\Rightarrow P(0) = P_0 = 2\ 000$

$$\Rightarrow P(t) = \frac{(10\ 000)(2\ 000)}{2000 + (10000 - 2000)\,e^{-at}}$$

$$= \frac{10\ 000}{1 + 4e^{-at}}$$

$$P(5) = 2500 = \frac{10\ 000}{1 + 4e^{-5a}}$$

$$\Rightarrow e^{-5a} = \frac{3}{4}$$

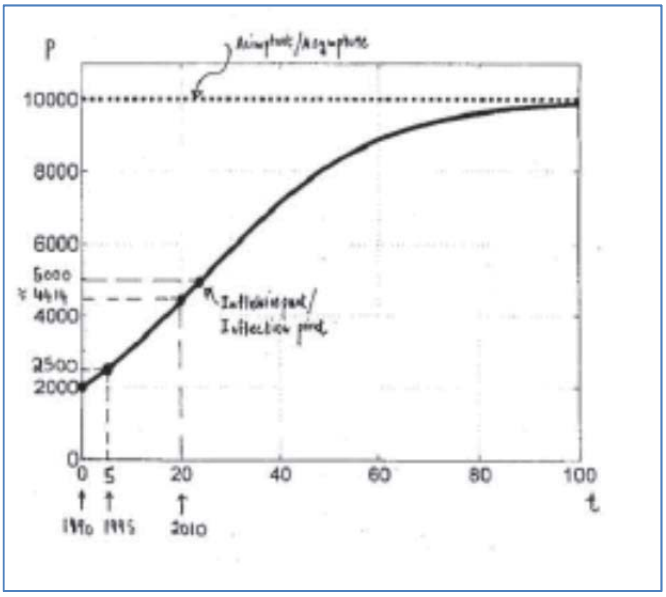
$$\Rightarrow e^{-a} = \left(\frac{3}{4}\right)^{1/5}$$

$$\Rightarrow e^{-at} = \left(\frac{3}{4}\right)^{t/5}$$

$$\Rightarrow P(t) = \frac{10\ 000}{1 + 4\left(\frac{3}{4}\right)^{t/5}}$$

Number of cats in 2010:

$$P(20) = \frac{10\ 000}{1 + 4\left(\frac{3}{4}\right)^{20/5}} \approx 4414 \text{ cats}$$



VRAAG 3 / QUESTION 3

Ex 2.8.; Nr $\boxed{3}$

$$\frac{dP}{dt} = P\left(10^{-1} - 10^{-7}P\right) \quad ; \quad P(0) = 5000$$

$$\Rightarrow a = 10^{-1} \; ; \; b = 10^{-7} \Rightarrow K = \frac{a}{b} = 10^{6} \text{ humans}$$

$$\Rightarrow P = \frac{10^{6}(5000)}{5000 + (10^{6} - 5000)\, e^{-10^{-1}t}} \leftarrow \text{from } 3$$

$$= \frac{10^{6}}{1 + 199\, e^{-t/10}}$$

$$t = t^{*} \text{ when } P = \frac{10^{6}}{2} = 500\ 000$$

$$\Rightarrow 500\ 000 = \frac{10^{6}}{1 + 199\, e^{-t^{*}/10}}$$

$$\Rightarrow t^{*} = 10 \ln 199 = 52,93 \text{ months}$$