TWB252

Tutoriaal 2	Tutorial 2
2 Augustus 2017	2 August 2017

U skryf 'n toets na afloop van die tutoriaal.

You write a test at the end of the tutorial.

1. 'n Pyl word met 'n kruisboog vertikaal na bo gelanseer met 'n aanvanklike snelheid van 49 m/s vanaf grondhoogte. Neem aan dat lugweerstand direk eweredig aan spoed is, en bepaal die maksimum hoogte wat die pyl sal bereik. Vergelyk hierdie hoogte met die maksimum hoogte wat bereik sou word as lugweerstand afwesig was.

Wenk: Herlei vanuit eerste beginsels 'n DV van die volgende vorm,

1. An arrow is shot vertically upwards with an initial velocity of 49 m/s from a crossbow at ground level. Assume that air resistance is proportional to speed, and determine the maximum height that the arrow will reach. Compare this height with the maximum height reached when air resistance is absent.

Hint: Derive from first principles a DE of the following form,

$$\frac{dv}{dt} = -\rho v - g,$$

en aanvaar dat $\rho = 0.04\,\mathrm{s}^{-1}$.

and assume that $\rho = 0.04\,\mathrm{s}^{-1}$.

Antw: 108.2 m as $R \propto v$; 122.5 m as $R = 0$

Ans: 108.2 m if $R \propto v$; 122.5 m if $R = 0$

2. Die aanwas van wildekatte op 'n afgeleë eiland word beperk deur onvoldoende kosvoorraad. Daar was 2000 katte in 1990, 2500 katte in 1995, en daar word beraam dat die bevolking nie 10000 kan oorskry nie. Verkry die bevolking as 'n funksie van tyd en beraam die aantal katte in 2010. Skets 'n grafiek van die bevolking waarop al die data hierbo plus asimptote en infleksiepunte aangedui word.

Antwoord: Getal katte in 2010 $\approx$ 4414.

2. The increase in numbers of a wild cat population on a remote island is limited by insufficient food supply. There were 2000 cats in 1990, 2500 cats in 1995, and it is estimated that the population can never exceed 10000. Find the population as a function of time, and estimate the number of cats in 2010. Sketch a graph of the population indicating the above data plus asymptotes and inflection points.

Answer: Number of cats in 2010 $\approx$ 4414.

3. Doen die volgende probleem in Oefening 2.8, Zill & Wright:

3. Do the following problem in Exercise 2.8, Zill & Wright:

Uitgawe/Edition 4	p. 84:	Nr. 3
Uitgawe/Edition 5	p. 88:	Nr. 3

(Maak ook 'n goeie vryhandskets van die oplossing.)

(Also make a good free hand sketch of the solution.)

Antwoorde: Dra-kapasiteit: 10^6 mense; Tydsduur: 52.93 maande.

Answers: Carrying capacity: 10^6 humans; Duration: 52.93 months.

4. Doen die volgende probleem in Oefening 2.8, Zill & Wright:

4. Do the following problem in Exercise 2.8, Zill & Wright:

Uitgawe/Edition 4	p. 84:	Nr. 5
Uitgawe/Edition 5	p. 88:	Nr. 5

In (b) is dit slegs nodig om die aanvangswaardeprobleem in (a) op te los.

In (b) it is only required to solve the initial-value problem in (a).

Antwoorde:

(b) $P(t) = \dfrac{4(P_0 - 1) - (P_0 - 4)\, e^{-3t}}{(P_0 - 1) - (P_0 - 4)\, e^{-3t}}$

(c) Wanneer $0 < P_0 < 1$ word die tyd wat dit sal neem vir die visbevolking om uit te sterf gegee deur $t = -\dfrac{1}{3} \ln \dfrac{4(P_0 - 1)}{P_0 - 4}$.

Answers:

(b) $P(t) = \dfrac{4(P_0 - 1) - (P_0 - 4)\, e^{-3t}}{(P_0 - 1) - (P_0 - 4)\, e^{-3t}}$

(c) When $0 < P_0 < 1$, the time that it will take for the fishery population to become extinct is given by $t = -\dfrac{1}{3} \ln \dfrac{4(P_0 - 1)}{P_0 - 4}$.

3. A model for the population $P(t)$ in a suburb of a large city is given by the initial-value problem

$$\frac{dP}{dt} = P(10^{-1} - 10^{-7}P), \quad P(0) = 5000,$$

where t is measured in months. What is the limiting value of the population? At what time will the population be equal to one-half of this limiting value?

5. **(a)** If a constant number h of fish are harvested from a fishery per unit time, then a model for the population $P(t)$ of the fishery at time t is given by

$$\frac{dP}{dt} = P(a - bP) - h, \quad P(0) = P_0,$$

where a, b, h, and P_0 are positive constants. Suppose $a = 5$, $b = 1$, and $h = 4$. Since the DE is autonomous, use the phase portrait concept of Section 2.1 to sketch representative solution curves corresponding to the cases $P_0 > 4$, $1 < P_0 < 4$, and $0 < P_0 < 1$. Determine the long-term behavior of the population in each case.

(b) Solve the IVP in part (a). Verify the results of your phase portrait in part (a) by using a graphing utility to plot the graph of $P(t)$ with an initial condition taken from each of the three intervals given.

(c) Use the information in parts (a) and (b) to determine whether the fishery population becomes extinct in finite time. If so, find that time.