## QUESTION 1

### Ex 2.7 : Nr 4

$$\frac{dP}{dt} = kP \;,\quad P(0) = P_0$$

$$\Rightarrow P = P_0 e^{kt}$$

$$400 = P_0 e^{3k} \ldots (1)$$
$$2000 = P_0 e^{10k} \ldots (2)$$

$$(2)/(1) : \quad 5 = e^{7k} \Rightarrow k = \frac{1}{7}\ln 5$$

Sub into (1) : 
$$400 = P_0 e^{3(\frac{1}{7}\ln 5)}$$
$$= P_0 \left(e^{\ln 5}\right)^{3/7}$$
$$= P_0 (5)^{3/7}$$

$$\Rightarrow P_0 = \frac{400}{(5)^{3/7}} \approx 201 \text{ bacteria}$$

### Ex 2.7 : Nr 10

$$\frac{dS}{dt} = rS \;;\quad S(0) = S_0$$

$$\Rightarrow S = S_0 e^{rt}$$

(a) $$S(5) = 5000\, e^{(0{,}0575)(5)}$$
$$= \$\, 6665{,}42$$

(b) $$2\cancel{S_0} = \cancel{S_0}\, e^{0{,}0575t}$$

$$2 = e^{0{,}0575t}$$

$$\Rightarrow t = \frac{\ln 2}{0{,}0575} = 12{,}05 \text{ years}$$

(c) $$S = \$\, 6651{,}82$$

### Ex 2.7 : Nr 11

$$N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/5600}$$

$$0{,}145 = \left(\frac{1}{2}\right)^{t/5600}$$

$$\ln 0{,}145 = \left(\frac{t}{5600}\right) \ln\left(\frac{1}{2}\right)$$

$$t = 5600 \left(\frac{\ln 0{,}145}{\ln 0{,}5}\right)$$
$$= 15\,600 \text{ years}$$

## Ex 2.7: Nr 15

Let $T = T(t)$ be the temperature of the rod at time $t$.

$$\frac{dT}{dt} = k(T - 100)$$

$$\frac{dT}{dt} - kT = -100k$$

Use integrating factor: I.F. $= e^{-kt}$

$$e^{-kt}\frac{dT}{dt} - ke^{-kt}T = -100ke^{-kt}$$

$$\frac{d}{dt}\left(e^{-kt}T\right) = -100ke^{-kt}$$

$$e^{-kt}T = 100e^{-kt} + C$$

$$T = 100 + Ce^{kt}$$

When $t = 0$, then $T = 20$:

$$20 = 100 + C$$

$$\Rightarrow C = -80$$

When $t = 1$, then $T = 22$:

$$22 = 100 - 80^{(1)(k)}$$

$$e^{k} = 78/80$$

$$k = \ln\left(39/40\right)$$

Therefore:

$$T = 100 - 80e^{(\ln 39/40)t}$$

$$= 100 - 80\left(39/40\right)^{t}$$

Consequently:

$$90 = 100 - 80\left(39/40\right)^{t}$$

$$\left(39/40\right)^{t} = 1/8$$

$$t\ln\left(39/40\right) = \ln\left(1/8\right)$$

$$\Rightarrow t = 82{,}13 \text{ seconds}$$

And:

$$98 = 100 - 80\left(39/40\right)^{t}$$

$$\left(39/40\right)^{t} = 1/40$$

$$t\ln\left(39/40\right) = \ln\left(1/40\right)$$

$$\Rightarrow t = 145{,}7 \text{ seconds}$$

## Ex 2.7. : Nr 21

Let $A = A(t)$ be the mass of salt (in grams) in the tank at time $t$ (in minutes).

$$\frac{dA}{dt} = (1)(4) - \left(\frac{A}{200}\right)(4)$$

$$= 4 - \frac{1}{50}A$$

$$\Rightarrow \frac{dA}{dt} + \frac{1}{50}A = 4$$

$$I.F. = e^{(1/50)t}$$

$$e^{(1/50)t}\frac{dA}{dt} + \frac{1}{50}e^{(1/50)t}A = 4e^{(1/50)t}$$

$$\frac{d}{dt}\left(e^{(1/50)t}A\right) = 4e^{(1/50)t}$$

$$e^{(1/50)t}A = 200e^{(1/50)t} + C$$

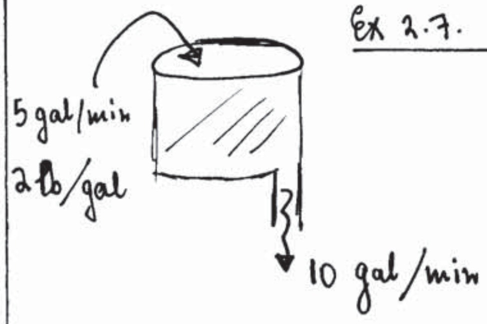
$$A = 200 + Ce^{-(1/50)t}$$

When $t = 0$, then $A = 30$:

$$30 = 200 + C \Rightarrow C = -170$$

$$\Rightarrow A(t) = 200 - 170e^{-(1/50)t}$$

## Ex 2.7. Nr 25




Inflow: 1 gal contains 2 lb salt

$\Rightarrow$ 5 gal " 10 lb salt

$\Rightarrow$ 10 lb/min

Outflow: After $t$ min there is $(500-5t)$ gal in tank which contains $A$ lb salt

$\Rightarrow$ 1 gal contains $\frac{A}{500-5t}$ lb salt

$\Rightarrow$ 10 gal " $\frac{10A}{500-5t}$ lb salt

$\Rightarrow \frac{10A}{500-5t} = \frac{2A}{100-t}$ lb/min

Rate of salt increase = Inflow − Outflow

$$\frac{dA}{dt} = 10 - \frac{2A}{100-t}$$

$$\frac{dA}{dt} + \left(\frac{2}{100-t}\right)A = 10$$

$$I.F. = e^{\int P(t)\,dt} = e^{2\int \frac{1}{100-t}dt} = e^{\ln|100-t|^{-2}}$$
$$= (100-t)^{-2}$$

$$\Rightarrow (100-t)^{-2}\frac{dA}{dt} + 2(100-t)^{-3}A = 10(100-t)^{-2}$$

$$\Rightarrow \frac{d}{dt}\left[(100-t)^{-2}A\right] = 10(100-t)^{-2}$$

$$(100-t)^{-2}A = 10(100-t)^{-1} + C$$

$$A(0) = 0 \Rightarrow C = -\frac{1}{10}$$

$$\Rightarrow A = 1000 - 10t - \frac{1}{10}(100-t)^2$$

Empty when $500 - 5t = 0 \Rightarrow t = 100$ min

QUESTION 2 (a)

$$\frac{dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = kN, \quad N(0) = N_0, \; k<0$$

or $k = -\lambda; \; \lambda > 0$

Separation of variables: $\int \frac{1}{N} dN = k \int dt + C$

$$\ln|N| = kt + C$$

$$|N| = De^{kt} \Rightarrow N = \pm De^{kt} = Ee^{kt}$$

$N(0) = E = N_0 \Rightarrow N(t) = N_0 e^{kt}$

Alternatively: Integrating factor

Half-life: $t_{1/2}$

$N(t_{1/2}) = \frac{1}{2}N_0 : \frac{1}{2}N_0 = N_0 e^{kt_{1/2}} \Rightarrow k = \frac{\ln(1/2)}{t_{1/2}}$

$\Rightarrow N = N_0 e^{[\ln(1/2)/t_{1/2}]t} = N_0 \left[e^{\ln(1/2)}\right]^{t/t_{1/2}} = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$

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(b) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}} \Rightarrow 0{,}95 = \frac{95}{100} = \left(\frac{1}{2}\right)^{t/1260}$

$$\ln(0{,}95) = \left(\frac{t}{1260}\right)\ln\left(\frac{1}{2}\right)$$

$\Rightarrow t = \frac{1260 \ln(0{,}95)}{\ln(1/2)} = 93{,}24$ million years