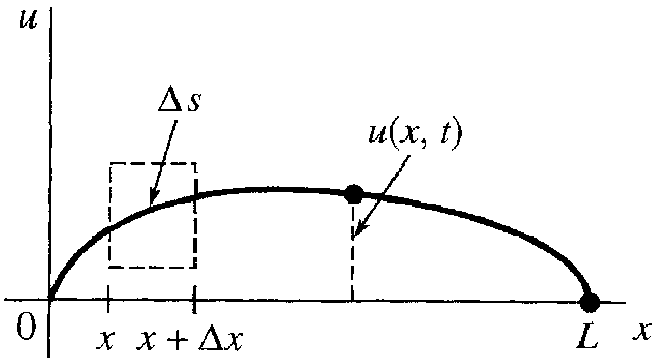
Die golfvergelyking

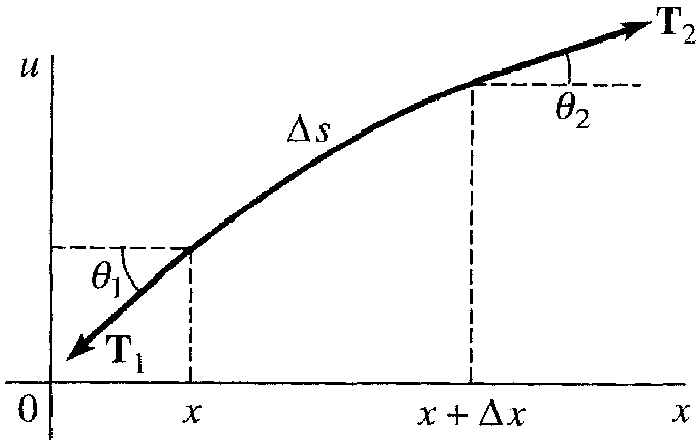
$$u_{tt} = a^2 u_{xx}$$

The wave equation

Herleiding/*Derivation* (p 694)



(a) segment of string



(b) enlargement of segment

Aannames:

(1) Die trekkrag in die snaar is orals dieselfde $\Rightarrow T_1 = T_2$

(2) Die uitwyking is klein

(3) Die snaar is uniform (massa/lengte (ρ) konstant)

(4) Die trekkrag T is groot $\Rightarrow$ swaartekrag kan geïgnoreer word

Assumptions:

(1) The tension in the string is the same everywhere $\Rightarrow T_1 = T_2$

(2) The deflection is small

(3) The string is uniform (mass/length (ρ) constant)

(4) The tension T is large $\Rightarrow$ gravity can be ignored

Newton se tweede wet / Newton's second law ($\Sigma F = ma$):

$$T_2 \sin\theta_2 - T_1 \sin\theta_1 = m\frac{\partial^2 u}{\partial t^2}$$

Klein uitwyking / Small deflection $\Rightarrow$

$$\sin\theta_1 \approx \tan\theta_1 \approx \frac{\partial u}{\partial x}(x,t)$$

$$\sin\theta_2 \approx \tan\theta_2 \approx \frac{\partial u}{\partial x}(x+\Delta x,t)$$

$$m = \rho\Delta x \quad \textbf{en/and} \quad T_1 = T_2 = T$$

$$\rho\Delta x\frac{\partial^2 u}{\partial t^2} = T\left(\frac{\partial u}{\partial x}(x+\Delta x,t) - \frac{\partial u}{\partial x}(x,t)\right)$$

$$\frac{\rho}{T}\frac{\partial^2 u}{\partial t^2} = \frac{\left(\frac{\partial u}{\partial x}(x+\Delta x,t) - \frac{\partial u}{\partial x}(x,t)\right)}{\Delta x}$$

As/*If* $\Delta x \to 0$ **dan**/*then*

$$\frac{\rho}{T}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad \Rightarrow \quad \frac{\partial^2 u}{\partial t^2} = \frac{T}{\rho}\frac{\partial^2 u}{\partial x^2}$$

Laat/*Let* $\dfrac{T}{\rho} = a^2$ **dan**/*then* $\boxed{u_{tt} = a^2 u_{xx}}$

Aanvangsvoorwaardes/*Initial conditions:*

$$u(x,0) = f(x) \quad \textbf{(aanvanklike posisie/}\textit{initial position}\textbf{)}$$
$$u_t(x,0) = g(x) \quad \textbf{(aanvanklike snelheid/}\textit{initial velocity}\textbf{)}$$

Randvoorwaardes/*Boundary conditions:*

$$u(0,t) = 0 \quad \textbf{(linker endpunt vas/}\textit{left endpoint fixed}\textbf{)}$$
$$u(L,t) = 0 \quad \textbf{(regter endpunt vas/}\textit{right endpoint fixed}\textbf{)}$$

(Voorbeeld/*Example*) **PDV/*PDE*:** $u_{tt} = a^2 u_{xx}, \quad a^2 = \dfrac{T}{\rho}$

Aanvangsvoorwaardes/*Initial conditions*:

$$\textbf{(1)}\ u(x,0) = \sin(\pi x)\ \ \textbf{(2)}\ u_t(x,0) = 0,\ x \in [0,1]$$

Randvoorwaardes/*Boundary conditions*:

$$\textbf{(1)}\ u(0,t) = 0\ \ \textbf{(2)}\ u(1,t) = 0$$

Oplossing/*Solution*: $u(x,t) = \cos(\pi a t)\sin(\pi x)$

Daar sal in die volgende tutoriaal van julle verwag word om hierdie oplossing te verkry / *It will be expected of you to obtain this solution in the next tutorial*

Toets solank dat die oplossing geldig is, maw dat dit die PDV, die rand- en -aanvangsvoorwaardes bevredig! / *Meanwhile, test the validity of the solution, ie verify whether it satisfies the PDE, the boundary conditions, and the initial conditions!*

Let op dat die periode van die beweging gegee word deur / *Note that the period of the motion is given by*

$$\frac{2\pi}{\pi a} = \frac{2}{a}$$

(interpreteer/*interpret*)

$$u(x,t) = \cos(\pi a t)\sin(\pi x)$$