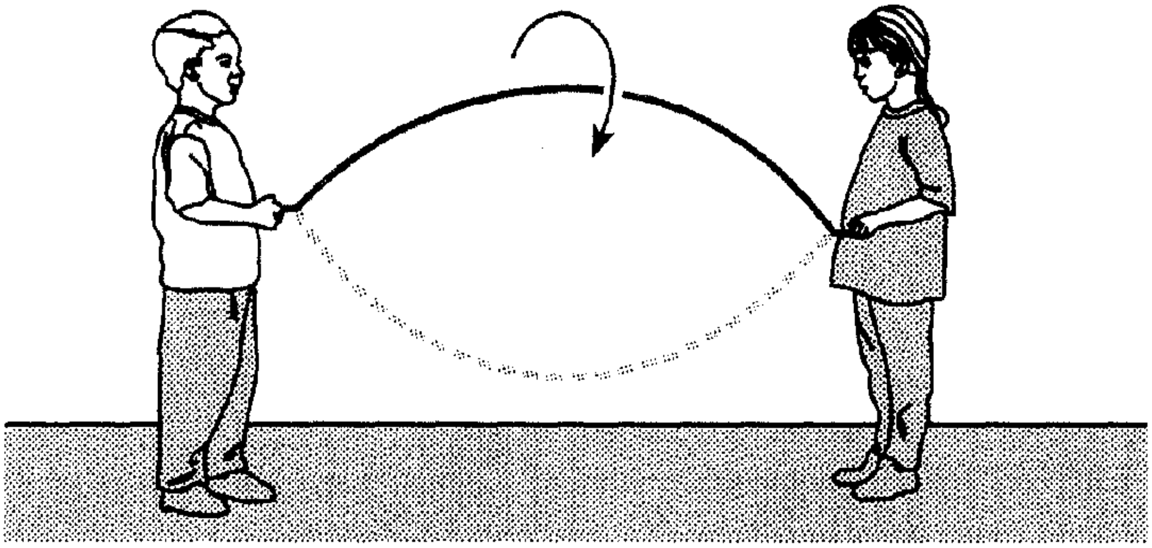
<u>3.9 Roterende tou</u> *(bl 169)*

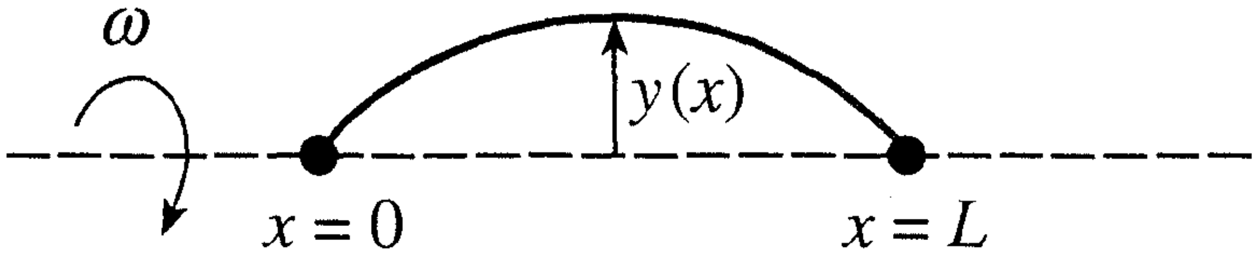
Probleem: Vir watter waarde(s) van die hoeksnelheid ω sal die tou deflekteer (uitwyk)?

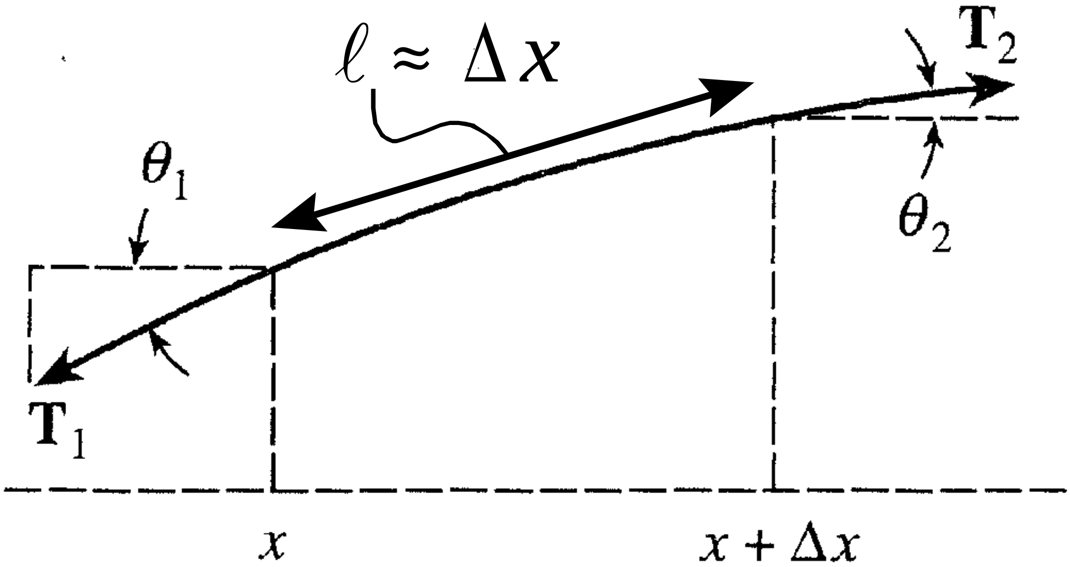


Aannames:

(1) Die hoeksnelheid ω is konstant
(2) Die trekkrag T in die tou is konstant en groot – gravitasie kan dus geïgnoreer word
(3) Die tou is uniform – die massa per eenheidslengte ρ is dus konstant
(4) Die uitwykings is klein
(5) Die endpunte is vas

Roterende tou   Tr 2





Newton se tweede wet: $(\downarrow)\,\Sigma F_y = ma$

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 = \overbrace{(\rho\,\Delta x)}^{m} \times \overbrace{(y\omega^2)}^{a=a_n}$$

$$\text{maar} \quad T_1 = T_2 = T$$

$$\rho\,\Delta x\, y\,\omega^2 = T(\sin\theta_1 - \sin\theta_2)$$

maar $\sin\theta_1 \approx \tan\theta_1$ en $\sin\theta_2 \approx \tan\theta_2$,

aangesien θ_1 en θ_2 klein is (klein uitwykings)

dus $\sin\theta_1 \approx y'(x)$ en $\sin\theta_2 \approx y'(x+\Delta x)$

$$\rho\, y\, \omega^2 = \frac{T(y'(x) - y'(x+\Delta x))}{\Delta x}$$

$$\rho\, y\, \omega^2 = -T\left(\frac{y'(x+\Delta x) - y'(x)}{\Delta x}\right)$$

maar $\displaystyle\lim_{\Delta x \to 0} \frac{y'(x+\Delta x) - y'(x)}{\Delta x} = y''(x)$

dus $\displaystyle\rho\, y\, \omega^2 = -T\frac{d^2y}{dx^2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{\rho\omega^2}{T}y = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \lambda y = 0 \quad \text{waar} \quad \lambda = \frac{\rho\omega^2}{T}$$

met randvoorwaardes $\quad y(0) = 0 \quad$ en $\quad y(L) = 0$

(Soos voorheen) Nie-triviale oplossings as:

$$\lambda_n = \frac{n^2\pi^2}{L^2} = \frac{\rho\,{\omega_n}^2}{T},\ n = 1, 2, \ldots$$

Dus kritieke hoeksnelhede: $\omega = \omega_n = \dfrac{n\pi}{L}\sqrt{\dfrac{T}{\rho}}$

Nie-triviale oplossings: $y_n = C\,\sin\left(\dfrac{n\pi x}{L}\right)$

$\boxed{\text{Vir } n = 1}$ Eerste kritieke hoeksnelheid

$$\omega_1 = \frac{\pi}{L}\sqrt{\frac{T}{\rho}}$$

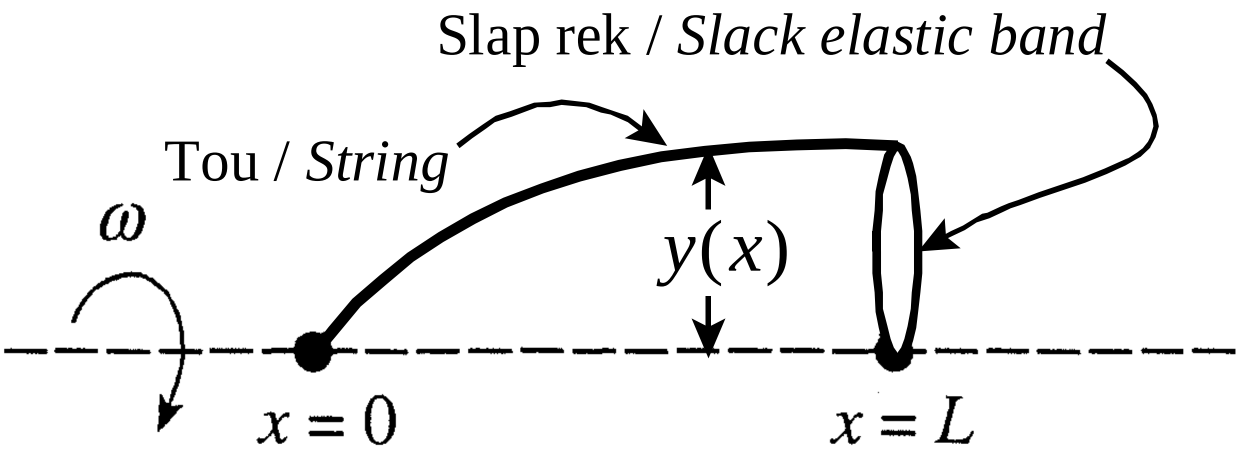
Eerste defleksie-modus: $y_1 = C\,\sin\left(\dfrac{\pi x}{L}\right)$

Roterende tou

$$\begin{array}{lll} 0 < \omega < \omega_1 & \Rightarrow & \text{geen defleksie} \\ \omega = \omega_1 & \Rightarrow & \text{eerste defleksie-modus} \\ \omega_1 < \omega < \omega_2 & \Rightarrow & \text{geen defleksie} \\ \omega = \omega_2 & \Rightarrow & \text{tweede defleksie-modus} \end{array}$$

Toon aan dat die eenheid vir ω rad/s is

Roterende tou met elastiese ondersteuning



$$\text{DV:} \quad \frac{d^2y}{dx^2} + \lambda y = 0 \quad \text{waar} \quad \lambda = \frac{\rho\omega^2}{T}$$

met randvoorwaardes $y(0) = 0$ en $y'(L) = 0$

Roteerende tou — Tr 6

Geval I: $\boxed{\lambda = 0}$ Toon aan $y \equiv 0$ (triviale opl)

Geval II: $\boxed{\lambda < 0}$ Toon aan $y \equiv 0$ (triviale opl)

Geval III: $\boxed{\lambda > 0}$ Toon aan...

Slegs nie-triviale oplossings as:

$$\lambda_n = \frac{(2n-1)^2}{L^2}\frac{\pi^2}{4},\ n = 1, 2, 3, \ldots$$

Nie triviale oplossings (eiefunksies):

$$y_n(x) = C\,\sin\left(\frac{\pi(2n-1)}{2L}\,x\right)$$

$\boxed{\text{Vir } n = 1}$ Eerste kritieke hoeksnelheid

$$\omega_1 = \frac{\pi}{2L}\sqrt{\frac{T}{\rho}}$$

Eerste defleksie-modus: $y_1 = C\,\sin\left(\frac{\pi x}{2L}\right)$ (skets)