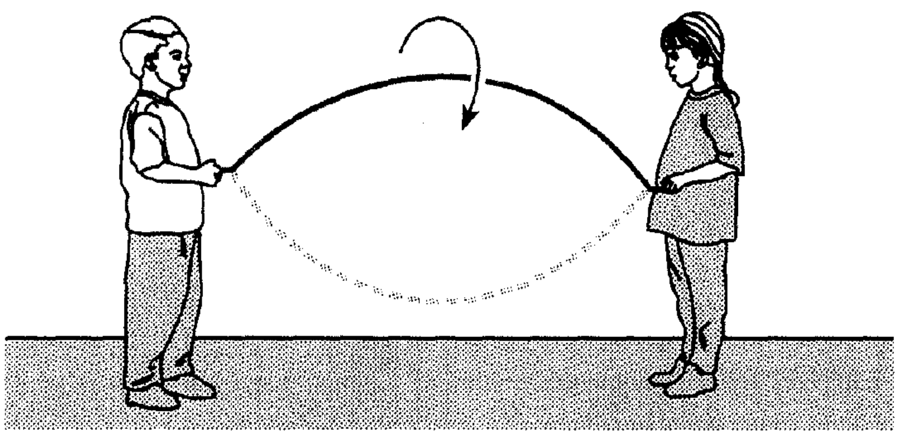
3.9 Roterende tou

(bladsy 169)

Probleem: Vir watter waarde(s) van die hoeksnelheid ω sal die tou deflekteer (uitwyk)?



3.9 Rotating string

(page 169)

Problem: For which value(s) of the angular velocity ω will the string (rope) be deflected?

Aannames:

(1) Die hoeksnelheid ω is konstant

(2) Die trekkrag T in die tou is konstant en groot. Gravitasie kan dus geïgnoreer word

(3) Die tou is uniform – die massa per eenheidslengte ρ is dus konstant

(4) Die uitwykings is klein

(5) Die endpunte is vas

Assumptions:

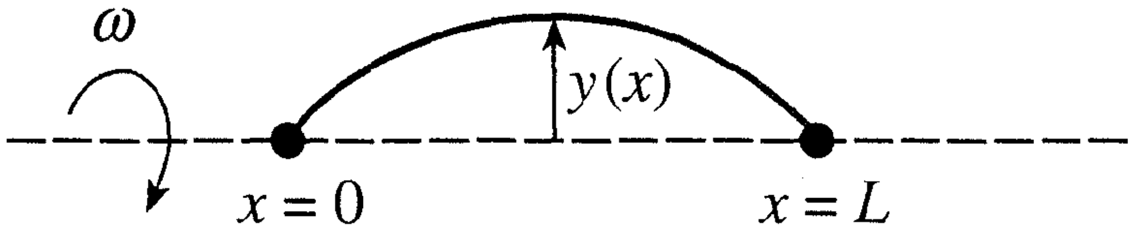
(1) The angular velocity ω is constant

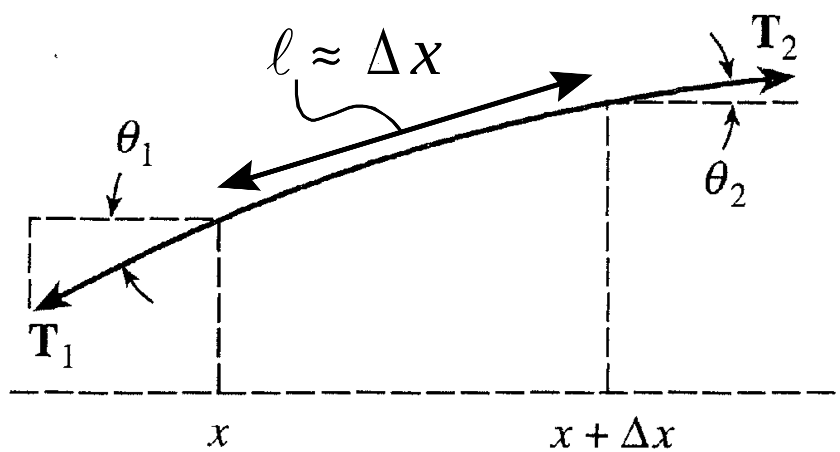
(2) The tension T in the rope is constant and large Gravitation can therefore be ignored

(3) The rope is uniform – the mass per unit length ρ is therefore constant

(4) The deflections are small

(5) The endpoints are fixed





Newton se tweede wet / *Newton's second law:* $(\downarrow)\, \Sigma F_y = ma$

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 = \overbrace{(\rho\, \Delta x)}^{\boxed{m}} \times \overbrace{(y\omega^2)}^{\boxed{a = a_n}}$$

maar/*but* $\quad T_1 = T_2 = T$

$$\rho\, \Delta x\, y\, \omega^2 = T(\sin\theta_1 - \sin\theta_2)$$

maar/*but* $\sin\theta_1 \approx \tan\theta_1$ **en/***and* $\sin\theta_2 \approx \tan\theta_2$,

aangesien θ_1 en θ_2 klein is (klein uitwykings)
since θ_1 and θ_2 are small (deflections small)

dus/*therefore* $\sin\theta_1 \approx y'(x)$ **en/***and* $\sin\theta_2 \approx y'(x+\Delta x)$

$$\rho\, y\, \omega^2 = \frac{T(y'(x) - y'(x+\Delta x))}{\Delta x}$$

$$\rho\, y\, \omega^2 = -T\left(\frac{y'(x+\Delta x) - y'(x)}{\Delta x}\right)$$

maar/*but* $\lim\limits_{\Delta x \to 0} \dfrac{y'(x+\Delta x) - y'(x)}{\Delta x} = y''(x)$

dus/*therefore* $\rho\, y\, \omega^2 = -T\dfrac{d^2y}{dx^2}$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{\rho\omega^2}{T}y = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \lambda y = 0 \quad \textbf{waar/}\textit{where} \quad \lambda = \frac{\rho\omega^2}{T}$$

met randvoorwaardes / *with boundary conditions* $y(0) = 0$ **en/***and* $y(L) = 0$

(Soos voorheen) Nie-triviale oplossings as/*(As before) Non-trivial solutions when:*

$$\lambda_n = \frac{n^2\pi^2}{L^2} = \frac{\rho\,{\omega_n}^2}{T}, \; n = 1, 2, \ldots$$

Dus kritieke hoeksnelhede/*Therefore critical angular velocities:* $\omega = \omega_n = \frac{n\pi}{L}\sqrt{\frac{T}{\rho}}$

Nie-triviale oplossings/*Non-trivial solutions:* $y_n = C\,\sin\left(\frac{n\pi x}{L}\right)$

Vir/*For* $n = 1$ **Eerste kritieke hoeksnelheid** / *First critical angular velocity*

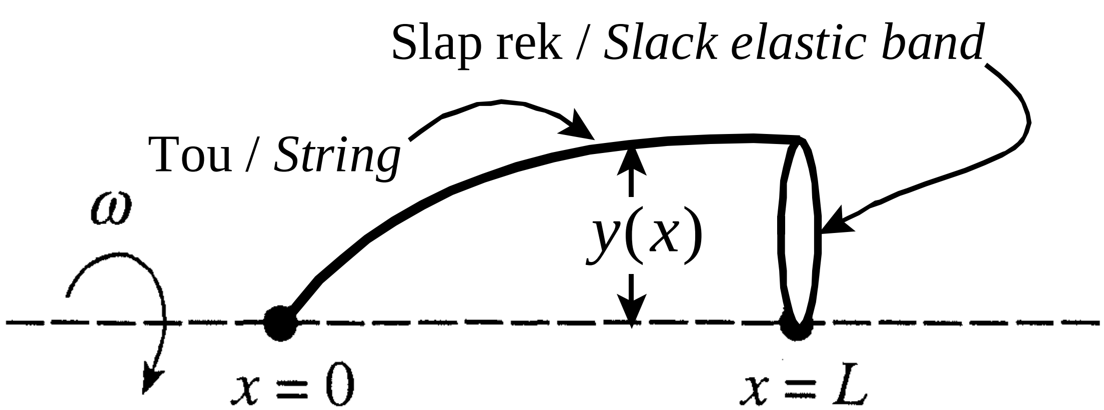
$$\omega_1 = \frac{\pi}{L}\sqrt{\frac{T}{\rho}}$$

Eerste defleksie-modus / *First deflection mode:* $y_1 = C\,\sin\left(\frac{\pi x}{L}\right)$

$0 < \omega < \omega_1 \Rightarrow$ **geen defleksie** / *no deflection*
$\omega = \omega_1 \Rightarrow$ **eerste defleksie-modus** / *first deflection mode*
$\omega_1 < \omega < \omega_2 \Rightarrow$ **geen defleksie** / *no deflection*
$\omega = \omega_2 \Rightarrow$ **tweede defleksie-modus** / *second deflection mode*

Toon aan dat die eenheid vir ω rad/s is / *Show that the unit for ω is rad/s*

Roterende tou met elastiese ondersteuning / *Rotating string with elastic support*



DV/*DE:* $\frac{d^2y}{dx^2} + \lambda y = 0$ **waar**/*where* $\lambda = \frac{\rho\omega^2}{T}$

met randvoorwaardes/*with boundary conditions* $y(0) = 0$ **en**/*and* $y'(L) = 0$

Geval I/*Case I:* $\boxed{\lambda = 0}$ **Toon aan**/*Show that* $y \equiv 0$ **(trivial(e) oplossing**/*solution***)**

Geval II/*Case II:* $\boxed{\lambda < 0}$ **Toon aan**/*Show that* $y \equiv 0$ **(trivial(e) oplossing**/*solution***)**

Geval III/*Case III:* $\boxed{\lambda > 0}$ **Toon aan dat...** / *Show that...*

Slegs nie-triviale oplossings as / *Non-trivial solutions only when:*

$$\lambda_n = \frac{(2n-1)^2}{L^2}\frac{\pi^2}{4}, \ n = 1, 2, 3, \ldots$$

Nie triviale oplossings (eiefunksies) / *Non-trivial solutions (eigenfunctions):*

$$y_n(x) = C \sin\left(\frac{\pi(2n-1)}{2L}x\right)$$

$\boxed{\textbf{Vir}/\textit{For } n = 1}$ **Eerste kritieke hoeksnelheid** / *First critical angular velocity*

$$\omega_1 = \frac{\pi}{2L}\sqrt{\frac{T}{\rho}}$$

Eerste defleksie-modus / *First deflection mode:* $y_1 = C \sin\left(\frac{\pi x}{2L}\right)$ **(skets**/*sketch***)**