2.7: Appl 2: Radioactive decay *(Page 73)*

Assumption: Rate of decay directly proportional to number of radioactive atoms present

Mathematical formulation:

Let $N = N(t)$ be the number of radioactive atoms at time t

$$\frac{dN}{dt} = -\lambda N \quad \text{with} \quad \lambda > 0$$

$$\lambda \equiv \text{decay constant} \quad \text{and} \quad N(0) = N_0$$

Halving time (half-life) $t_{1/2}$ (from previous work):

$$t_{1/2} = \frac{\ln 2}{\lambda} \ \text{ or } \ \lambda = \frac{\ln 2}{t_{1/2}}$$

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

- $C^{14} \to N^{14}$: $t_{1/2} = 5600$ years (C-14 test)
- $Ra^{226} \to Rn^{222}$: $t_{1/2} = 1700$ years
- $U^{238} \to Pb^{206}$: $t_{1/2} = 4.5$ billion years!

(Radioactive → Not) Large $t_{1/2} \Rightarrow$ Stable isotope

C-14 test ("Radiocarbon dating"):

- Small amounts radioactive C^{14} in atmosphere
- Plants/animals take up radioactive C^{14}
- $\Rightarrow C^{14}$ in plants/animals $\approx C^{14}$ in atmosphere
- Plant/animal dies: C^{14} uptake stops & $C^{14} \to N^{14}$
- Can now estimate age of fossil (t) with:

$$\frac{N(t)}{N_0} = \left(\frac{1}{2}\right)^{t/5600}, \quad t \text{ in years}$$

- N_0 known (as in atmosphere)
- $N(t)$ measured with Geiger counter

Only accurate to age of $60\,000$ years

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Example: A piece of fossilized wood contains 63% as much C^{14} as living wood with the same mass. How old is the fossilized wood?

Answer: 3733 years