2.7: Application 1: Population growth

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Problem: Predict the size of a population at any time t in the future

Assumptions:
- Population P continuous (smooth) function of time t: $P = P(t)$
- Rate of increase in P is directly proportional to size of P
- Limitless resources

Mathematical formulation:

$$\frac{dP}{dt} = kP \quad \text{with} \quad P(0) = P_0$$

Solution: $P = P_0 e^{kt}$ or $P = P_0 (2)^{t/t_2}$

$$k \equiv \text{growth constant} \quad (k > 0)$$

Incorporation of birth and death rate, b and s:

$$\frac{dP}{dt} = bP - sP = \underbrace{(b - s)}_{k} P$$

Population growth — Tr 2

Example: Cells in a river grow at a rate directly proportional to the number of cells present. After one hour, 1000 cells are counted and after 4 hours, 3000 cells.

Determine:

(a) the growth constant

(b) the initial number of cells

(c) the doubling time

(d) the number of cells at any time t

Let $P = P(t)$ be number of cells at time t (in hours)

Answers:

(a) $k = \frac{\ln 3}{3} = 0.3662$

(b) $P_0 = \frac{1000}{3^{1/3}} = 693.4$ cells

(c) $t_2 = \frac{3 \ln 2}{\ln 3} = 1.893$ hours

(d) $P(t) = 693.4e^{0.3662t}$ or $P(t) = 693.4(2)^{t/1.893}$

Population growth Tr 3

Problem 1 (p 78): The population of a community is known to increase at a rate proportional to the number of people present at time t. If an initial population P_0 has doubled in 5 years, how long will it take to triple? To quadruple?

Answers:

To triple:	7.9 years
To quadruple:	10 years