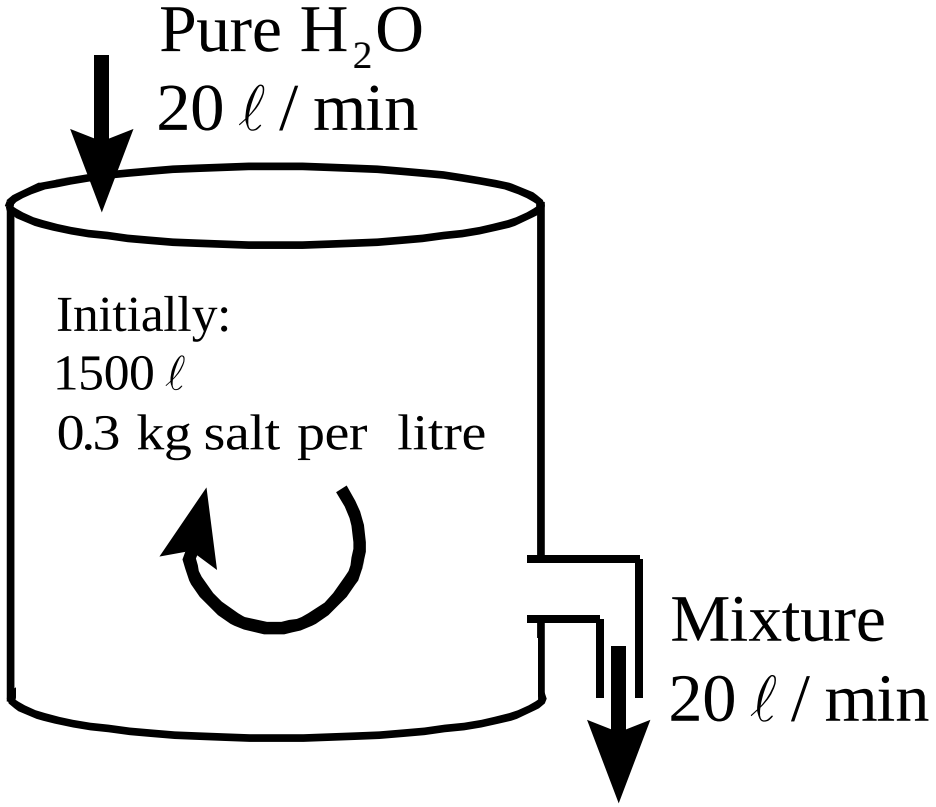
2.7: Application 5: Mixtures *(p 21, 76, 77)*

Example 1: A tank (with 1500ℓ salt water) contains a concentration of 0.3 kg salt per litre. In order to dilute the concentration, pure water is added at $20\ \ell$/minute. The mixture is stirred well and exits the tank at $20\ \ell$/minute. Determine the concentration salt in the tank at any time t.



Assumption:

Rate at which mass of salt in tank **increases**	=	Rate at which mass of salt **enters** tank	−	Rate at which mass of salt **exits** tank

Mathematical formulation:

Let $m = m(t)$ be the mass of the salt in the tank at time t (in minutes)

$$\frac{dm}{dt} = 0 - \frac{m}{1500}(20)$$

(make sure that the units, left & right, correspond)

$$\Rightarrow \quad \frac{dm}{dt} = -\frac{1}{75}m$$

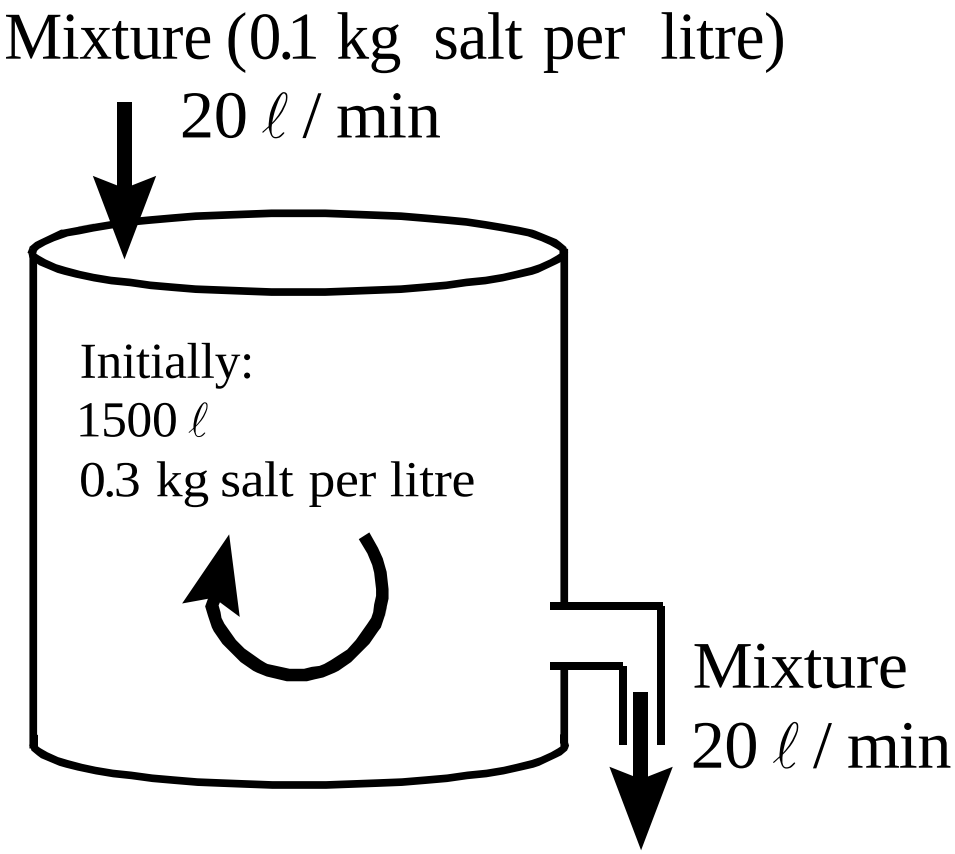
$$m(0) = m_0 = 0.3\text{kg}/\ell \times 1500\ell = 450\text{kg}$$

$$m(t) = 450e^{-\frac{1}{75}t}$$

Concentration: $c(t) = m(t)/1500$

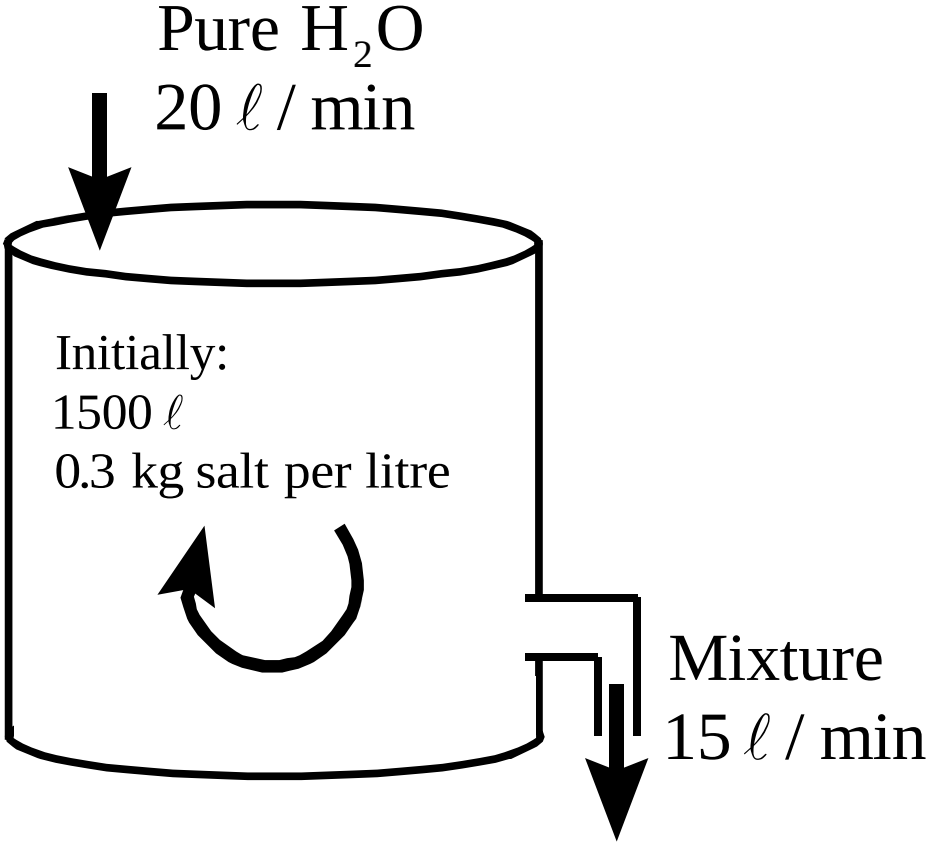
$$c(t) = \frac{450}{1500}e^{-\frac{1}{75}t} = 0.3e^{-\frac{1}{75}t}$$

Example 2: A tank (with 1500ℓ salt water) contains a concentration of 0.3 kg salt per litre. In order to dilute the concentration, salt water (with a concentration of 0.1 kg salt per litre) is added at $20\ \ell$/minute. The mixture is stirred well and exits the tank at $20\ \ell$/minute. Determine the concentration salt in the tank at any time t.

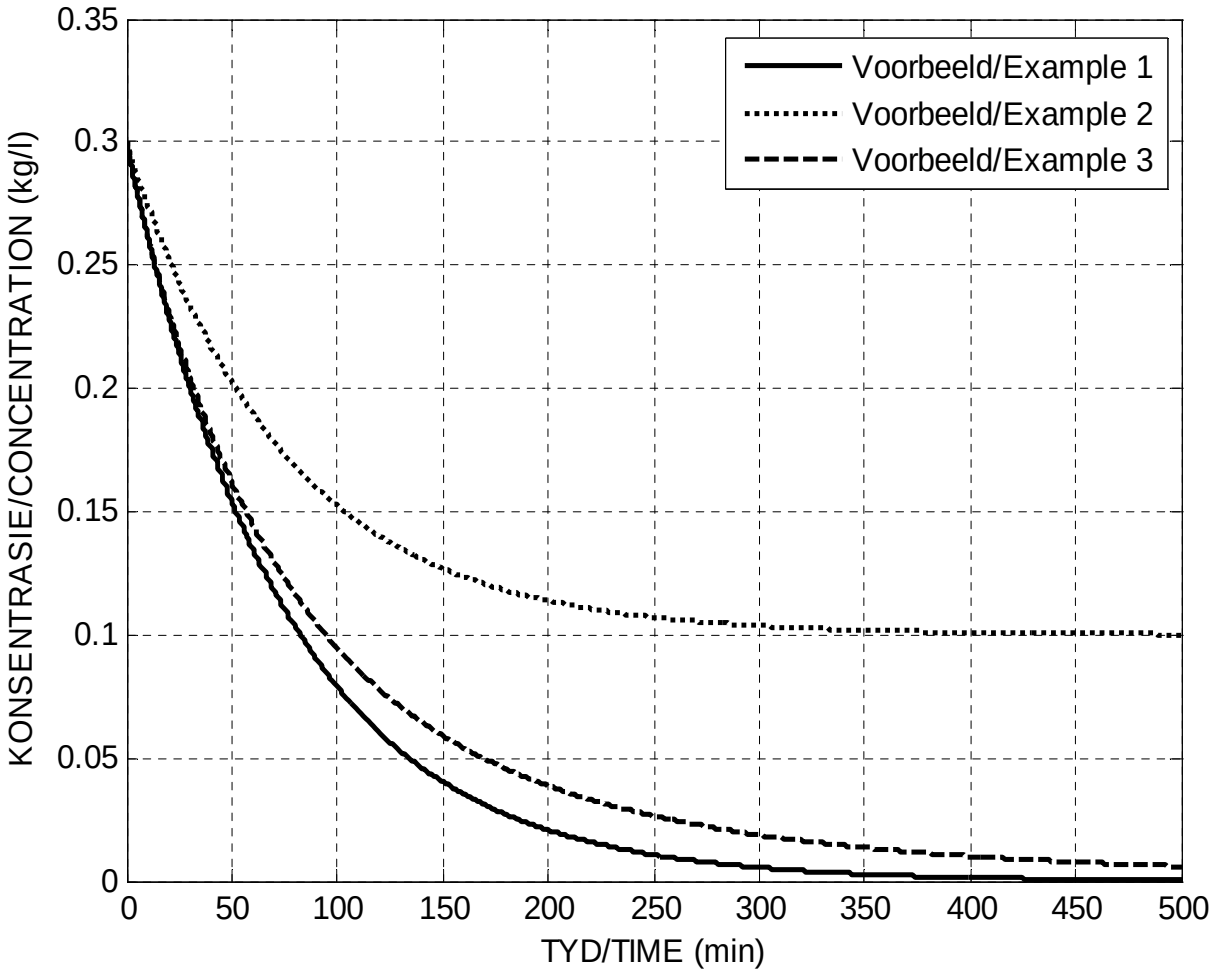


Ans: $c(t) = 0.1 + 0.2e^{-\frac{1}{75}t}$

Example 3: A tank (with 1500ℓ salt water) contains a concentration of 0.3 kg salt per litre. In order to dilute the concentration, pure water is added at $20\ \ell$/minute. The mixture is stirred well and exits the tank at $15\ \ell$/minute. Determine the concentration salt in the tank at any time t.



Ans: $c(t) = \dfrac{(90)(300^3)}{(300+t)^4}$



(SELF STUDY) **Application 6:** Electrical circuits *(p 22, 77, 78)*