2.8: Nie-lineêre modelle (bl 83)

2.8: Non-linear models (p 83)

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Bevolkingsgroei / *Population growth*

Maltus-model (lineêr / *linear*): $\dfrac{dP}{dt} = kP, \; P(0) = P_0$

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Inkorporering van geboortes (a) en sterftes (b):
Incorporation of births (a) and deaths (b):

$$\frac{dP}{dt} = aP - bP = \underbrace{(a-b)}_{k} P$$

Geboortetempo $\equiv aP$**; Sterftetempo** $\equiv bP$
Birth rate $\equiv aP$*; Death rate* $\equiv bP$

Steeds eksp groei as $a > b$ **& eksp verval as** $a < b$
Still exp growth if $a > b$ *& exp decay if* $a < b$

Dus nie 'n verbetering nie! / *Therefore not an improvement!*

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Verhulst se verbetering: Logistiese model /
Verhulst's improvement: Logistic model

Verbeterde aanname: Hoe groter die bevolking (P**), hoe groter die sterftetempo (**bP**), weens kompetisie**
Improved assumption: The larger the population (P*), the larger the death rate (*bP*), due to competition*

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Improved assumption: The larger the population (P)*, the larger the death rate* (bP)*, due to competition*

Model: $$\frac{dP}{dt} = aP - (bP)P = aP - bP^2$$

Herskryf /*Re-written:* $$\boxed{\frac{dP}{dt} = P(a - bP),\ P(0) = P_0}$$

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NB: **As/** *If* $P = \dfrac{a}{b} = K$ **dan/** *then* $\dfrac{dP}{dt} = 0 \Rightarrow P$ **stabiel/** *stable*

$K \equiv$ **Drakapasiteit van omgewing/** *Carrying capacity of environment*

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Oplossing / *Solution:* **Integrasiefaktore gaan nie werk nie, want DV is nie-lineêr** / *Integration factors will not work, since DE is non-linear*

Gebruik dus skeiding van veranderlikes en parsiële breuke, en toon aan dat / *Therefore use separation of variables and partial fractions, and show that*:

$$\boxed{P(t) = \frac{\frac{a}{b}P_0}{P_0 + (\frac{a}{b} - P_0)e^{-at}}}$$

NB: **As/***If* $t = 0$ **dan/***then* $P = P_0$

As/*If*: $t \to \infty$ **dan/***then* $P \to \dfrac{a}{b} = K$ **(drakapasiteit/***carrying capacity***)**

NB: **As/***If* $t = 0$ **dan/***then* $P = P_0$
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Voorbeeld: **Die bevolkingsdinamika van 'n kolonie bakterieë kan gemodeleer word met Verhulst se logistiese groeimodel. Die kolonie bestaan aanvanklik uit ongeveer** $100\,000$ **individue en na** 7 **uur word ongeveer** $200\,000$ **individue waargeneem. Uiteindelik blyk dit dat die bevolking op ongeveer** $1\,000\,000$ **individue stabiliseer.** / *Example: The population dynamics of a colony of bacteria can be modelled by Verhulst's logistic growth model. The colony initially consists of approximately* $100\,000$ *individuals and approximately* $200\,000$ *individuals are counted* 7 *hours later. Eventually it appears that the population stabilizes at approximately* $1\,000\,000$ *individuals.*

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Verkry nou 'n skatting vir die bevolking op enige tydstip t**.** / *Now find an estimation for the population at any time* t*.*

Laat $P = P(t)$ **die aantal bakterieë (in duisende) op tyd** t **(in uur) wees** / *Let* $P = P(t)$ *be the number of bacteria (in thousands) at time* t *(in hours)*

NB: **As**/*If* $t = 0$ **dan**/*then* $P = P_0$

As/*If*: $t \to \infty$ **dan**/*then* $P \to \dfrac{a}{b} = K$ (**drakapasiteit**/*carrying capacity*)

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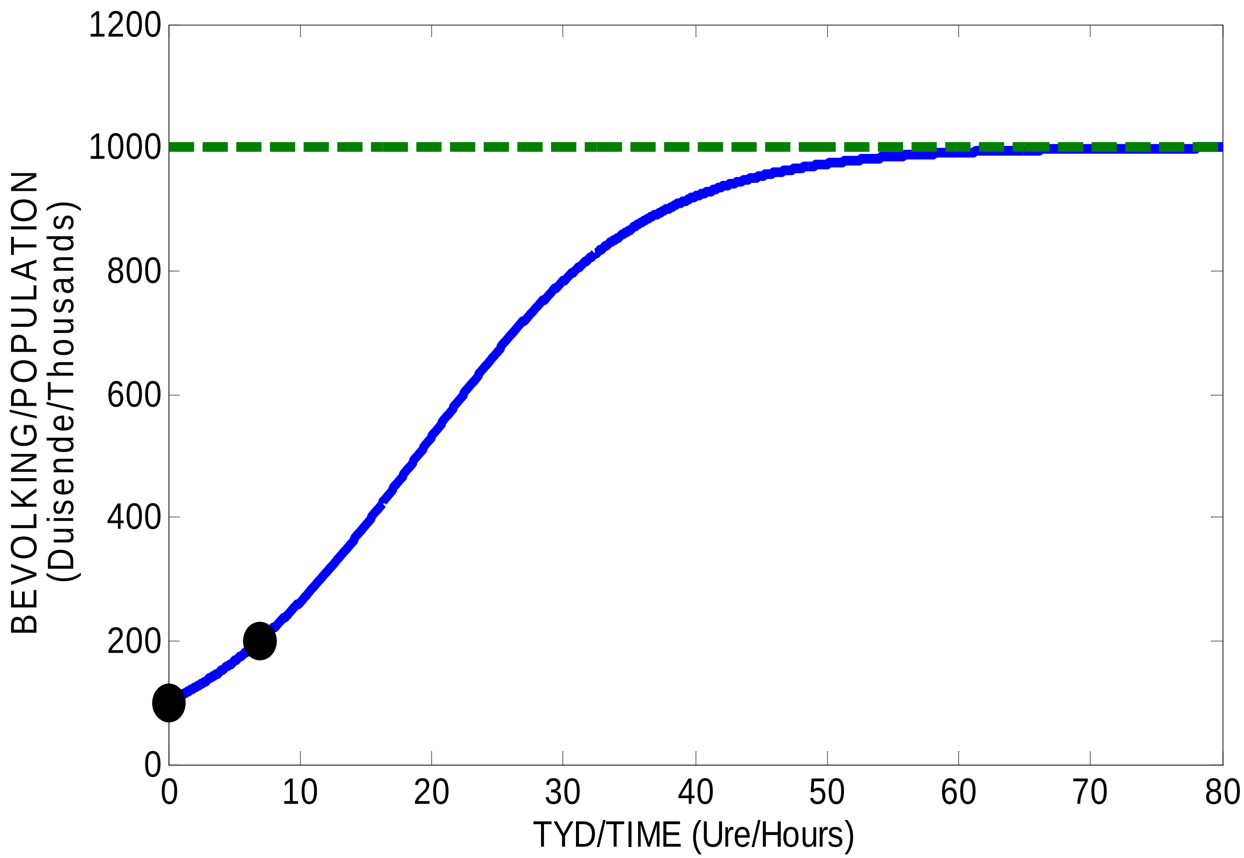
Antwoord (in duisende) / *Answer (in thousands):* $P(t) = \dfrac{1000}{1 + 9(4/9)^{t/7}}$



Grafiek vir algemene geval: sien Fig 2.8.2 (bl 84)

Graph for general case: see Fig 2.8.2 (page 84)

SELFSTUDIE: (bl 85)
- **Modifikasies van logistiese vgl**
- **Chemiese reaksies**

SELF STUDY: (page 85)
- *Modifications of logistic equation*
- *Chemical reactions*