2.7: Lineêre modelle (bl 72)

2.7: Linear models (p 72)

2.7: Lineêre modelle (bl 72)

Eksponensiële groei/verval: **(Baie belangrike DV!)**

$$\frac{dx}{dt} = kx, \text{ waar } k \text{ konstante; Aanvangswaarde: } x(0) = x_0$$

Oplossingsmetodes: **(a) Skeiding van veranderlikes**
(b) Integrasiefaktor

2.7: Linear models (p 72)

Exponential growth/decay: **(Very important DE!)**

$$\frac{dx}{dt} = kx, \text{ where } k \text{ constant; Initial condition: } x(0) = x_0$$

Solution methods: **(a) Separation of variables**
(b) Integrating factor

(a) Skeiding van veranderlikes / *Separation of variables*

$$\int \frac{dx}{x} = k \int dt + C_1$$

$$\ln|x| = kt + C_1$$

$$|x| = e^{kt}e^{C_1} = C_2e^{kt}$$

$$x = \pm C_2e^{kt} = C_3e^{kt}$$

Aanvangswaarde / *Initial condition:* $x(0) = x_0 = C_3$

Dus / *Therefore:* $\boxed{x(t) = x_0e^{kt}}$

(Memoriseer oplossing!) *(Memorize solution!)*

(b) Integrasiefaktor / *Integrating factor* *(I.F.)*

$$\frac{dx}{dt} \underbrace{-k}_{P(t)} x = 0 \tag{1}$$

Vermenigvuldig / *Multiply* **(1) met** / *with* $I.F. = e^{\int P(t)dt} = e^{-kt}$**:**

$$e^{-kt}\frac{dx}{dt} - ke^{-kt}x = 0$$

$$\frac{d}{dt}(e^{-kt}x) = 0$$

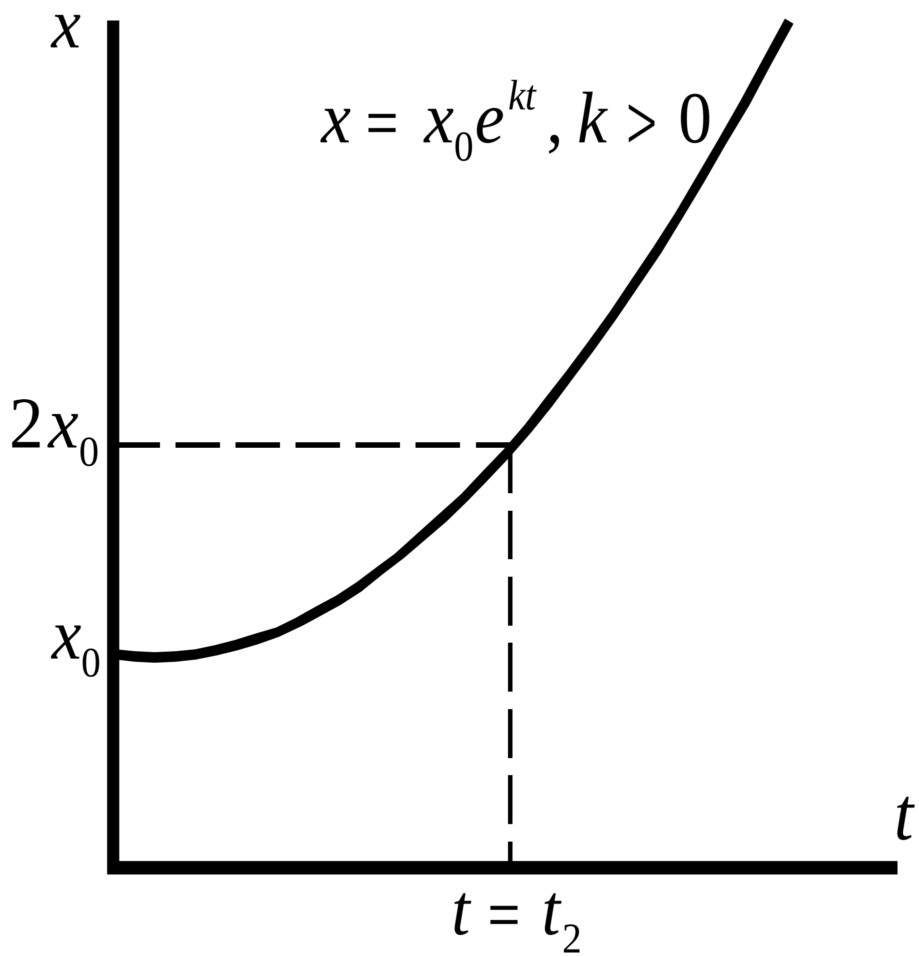
$$e^{-kt}x = C$$

Soortgelyk aan (a) is / *Similar to (a) we have:* $C = x_0$

Dus / *Therefore:* $\boxed{x(t) = x_0 e^{kt}}$

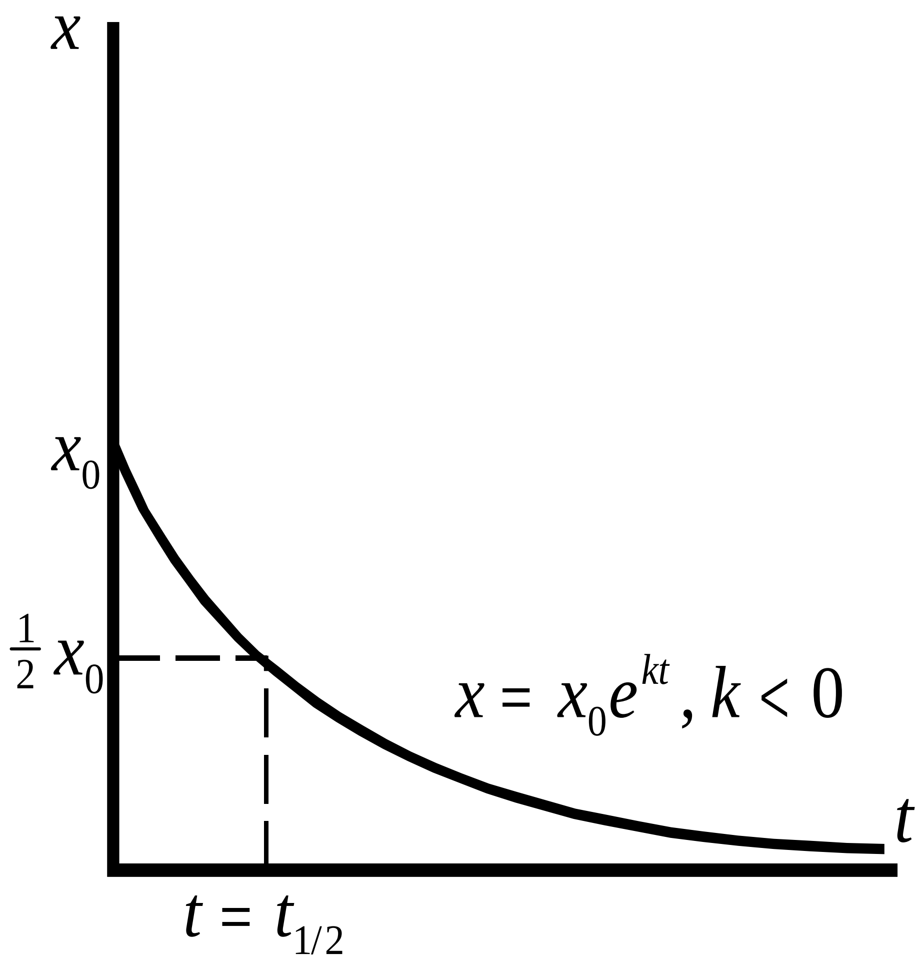
Eksponensiële groei ($k > 0$): $t_2 \equiv$ verdubbelingstyd

Exponential growth ($k > 0$): $t_2 \equiv$ doubling time



Eksponensiële verval ($k < 0$): $t_{1/2} \equiv$ halveertyd

Exponential decay ($k < 0$): $t_{1/2} \equiv$ halving time



Verdubbelingstyd (in geval van eksponensiële groei)

Doubling time (in the case of exponential growth)

$$x(t) = x_0 e^{kt}, \; k > 0$$

As / *If* $x = 2x_0$ **dan** / *then* $t = t_2$:

$$\Rightarrow \quad 2x_0 = x_0 e^{kt_2}$$

$$\Rightarrow \quad \boxed{t_2 = \frac{\ln 2}{k}} \quad \textbf{en} \; / \textit{and} \quad k = \frac{\ln 2}{t_2}$$

LW: • **Hoe groter k, hoe kleiner t_2, en omgekeerd**
• **t_2 is onafhanklik van x_0**

Note: • *The larger k, the smaller t_2, and vice versa*
• *t_2 is independent of x_0*

Alternatiewe formule vir die oplossing:

Alternative formula for the solution:

$$
\begin{aligned}
x(t) &= x_0 e^{kt}, \ k > 0 \\
&= x_0 e^{\frac{t \ln 2}{t_2}} \\
&= x_0 (e^{\ln 2})^{t/t_2} \\
&= x_0 (2)^{t/t_2}
\end{aligned}
$$

Halveertyd (in geval van eksponensiële verval)

Halving time (in the case of exponential decay)

$$x(t) = x_0 e^{kt},\ k < 0$$

As / *If* $x = \frac{1}{2}x_0$ **dan** / *then* $t = t_{1/2}$:

$$\Rightarrow \quad \frac{1}{2}x_0 = x_0 e^{kt_{1/2}}$$

$$\Rightarrow \quad \boxed{t_{1/2} = \frac{-\ln 2}{k}} \quad \textbf{en} \ / \textit{and} \quad k = \frac{-\ln 2}{t_{1/2}}$$

Alternatiewe formule vir die oplossing / *Alt formula for the solution:*

$$\begin{aligned} x(t) &= x_0 e^{kt},\ k < 0 \\ &= x_0 e^{-\frac{t \ln 2}{t_{1/2}}} = x_0 (e^{-\ln 2})^{t/t_{1/2}} = x_0 \left(\frac{1}{2}\right)^{t/t_{1/2}} \end{aligned}$$

Lineêre modelle / *Linear models*

Toepassings	**Applications**
(1) *Bevolkingsgroei*	**(1)** *Population growth*
(2) *Radioaktiewe verval*	**(2)** *Radioactive decay*
(3) *Saamgestelde rente*	**(3)** *Compound interest*
(4) *Newton se wet van afkoeling* (selfstudie)	**(4)** *Newton's law of cooling* (self-study)
(5) *Mengsels*	**(5)** *Mixtures*
(6) *Elektriese stroombane* (selfstudie)	**(6)** *Electrical circuits* (self-study)
(7) *Vryval teen lugweerstand*	**(7)** *Free-fall against air resistance*