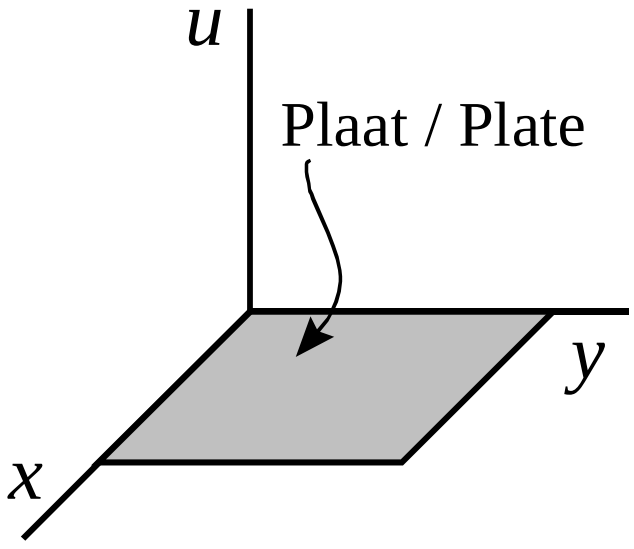
Laplace's equation $u_{xx} + u_{yy} = 0$

"Derivation" *(page 694)*

The two-dimensional heat equation is as follows:

$$u_t = k(u_{xx} + u_{yy}) \tag{1}$$

The initial temperature throughout the plate, that is $u(x, y, 0)$, and the temperature on the edge of the plate (for all t) are known; we now seek the temperature throughout the plate (for all t), that is $u = u(x, y, t)$

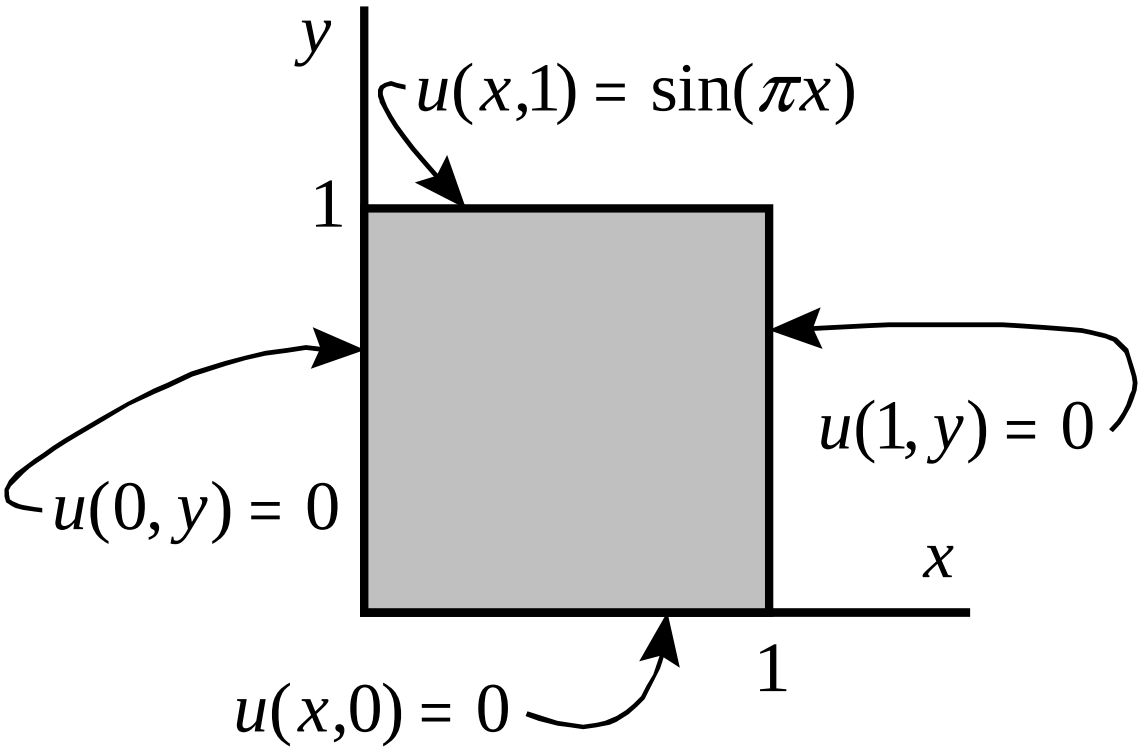


If we are only interested in the (eventual) steady-state temperature (that is where $u_t = 0$), equation (1) simplifies to

$$u_{xx} + u_{yy} = 0,$$

so that it is no longer required to specify k or the initial temperature throughout the plate $--$ the temperature on the edge of the plate (for all t) is sufficient!

Example



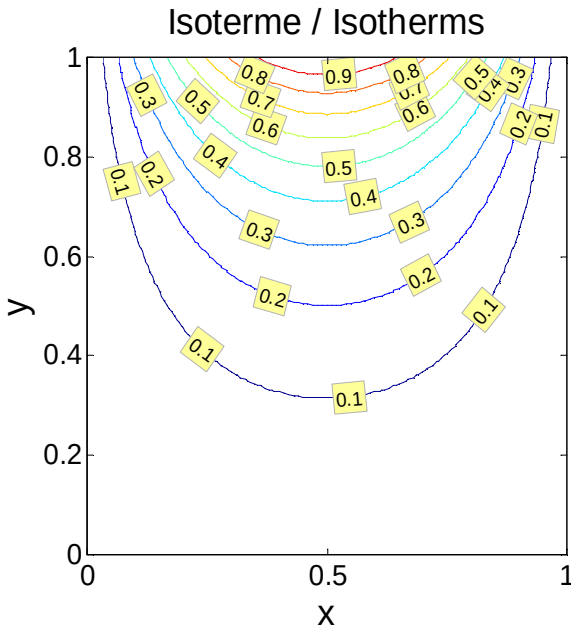
Laplace's equation Tr 3

Solution: $u(x,y) = \dfrac{\sinh(\pi y)}{\sinh(\pi)} \cdot \sin(\pi x)$

It will be expected of you to obtain this solution in the next tutorial

Meanwhile, test the validity of the solution - does it satisfy the PDE and the boundary conditions?!

Isotherms



Surface plot