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$$m\frac{dv}{dt} = mg - kv \quad \Rightarrow \quad \boxed{v(t) = \frac{mg}{k}\left(1 - e^{-kt/m}\right)}$$

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Met lugweerstand/*With air resistance* **(Nie-lineêre**/*Non-linear* **model:** $R \propto v^2$**)**

$$m\frac{dv}{dt} = mg - kv^2 \quad \Rightarrow \quad \boxed{v(t) = \textbf{?}}$$

Eenheid van k / *Unit of k:* $[k] = \mathsf{kg/m}$

Herskryf DV / *Re-write DE:* $\dfrac{dv}{dt} = -\dfrac{k}{m}(v^2 - a^2)$, *waar / where* $a^2 = \dfrac{mg}{k}$

Skeiding van veranderlikes / *Separation of variables:*

$$\frac{1}{2a}\left(\ln|v-a| - \ln|v+a|\right) = -\frac{k}{m}t + C$$

Aanvangsvoorwaarde / *Initial condition:* $v(0) = 0 \quad \Rightarrow \quad C = 0$

$$\ln\left|\frac{v-a}{v+a}\right| = -\frac{2ak}{m}t$$

$$\frac{ak}{m} = \sqrt{\frac{gk}{m}}$$

$$\left|\frac{v-a}{v+a}\right| = e^{-2\sqrt{\frac{gk}{m}}t}$$

$$\frac{v-a}{v+a} = \pm e^{-2\sqrt{\frac{gk}{m}}t}$$

Wanneer $t = 0$ en $\boxed{+}$ gekies word, dan $a = 0 \Rightarrow m = 0 \Rightarrow$ teenstrydigheid

When $t = 0$ and $\boxed{+}$ is chosen, then $a = 0 \Rightarrow m = 0 \Rightarrow$ contradiction

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Maak v die onderwerp van die vergelyking / *Make v the subject of the equation:*

$$v(t) = a\left(\frac{1 - e^{-2\sqrt{\frac{gk}{m}}t}}{1 + e^{-2\sqrt{\frac{gk}{m}}t}}\right)$$

Let op / *Note:* $v \to a$ **as / *if*** $t \to \infty$

$\Rightarrow$ **Limietsnelheid / *Limiting velocity:*** $a = \sqrt{\dfrac{mg}{k}}$ **(bevestig eenheid / *verify unit*)**

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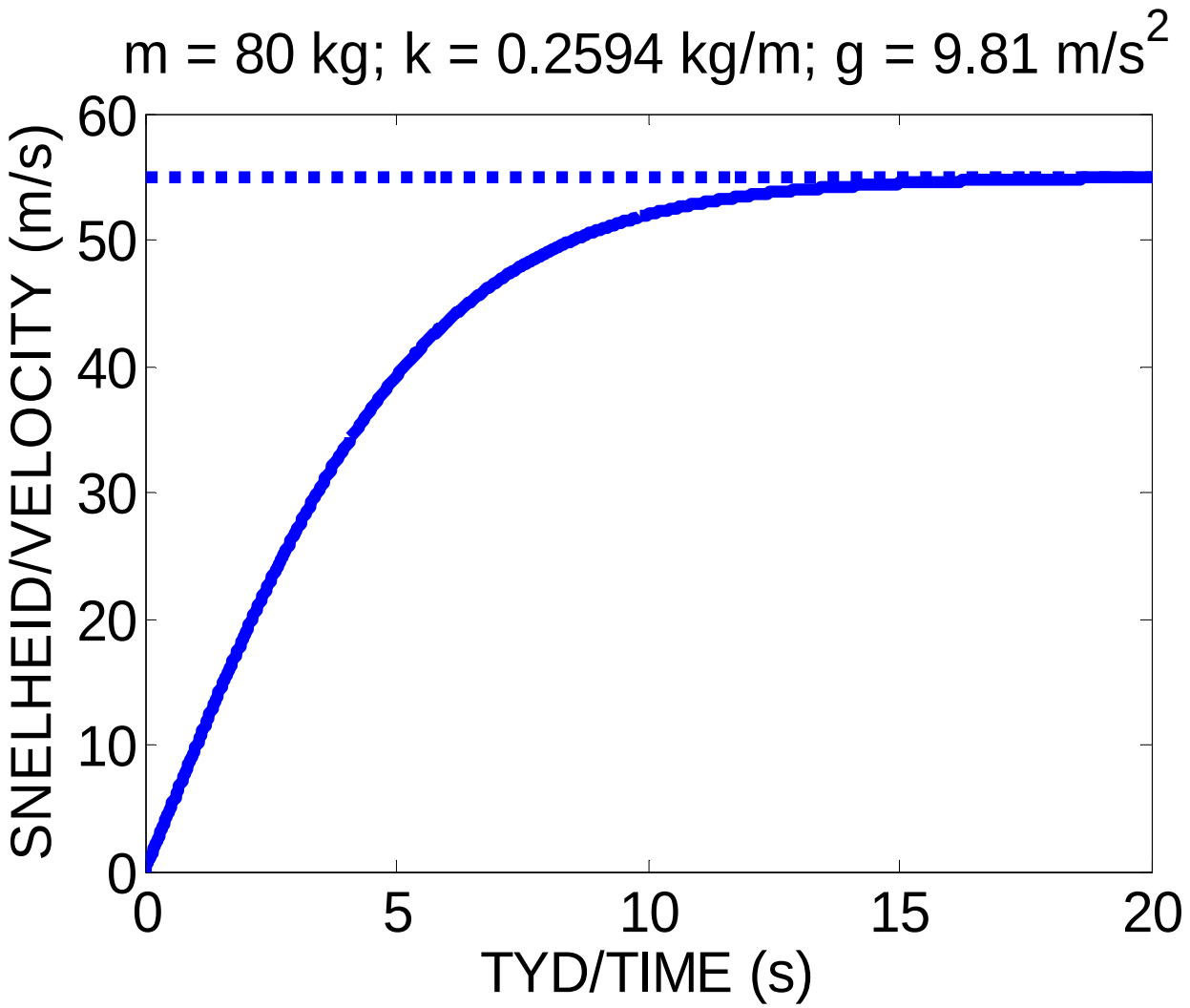
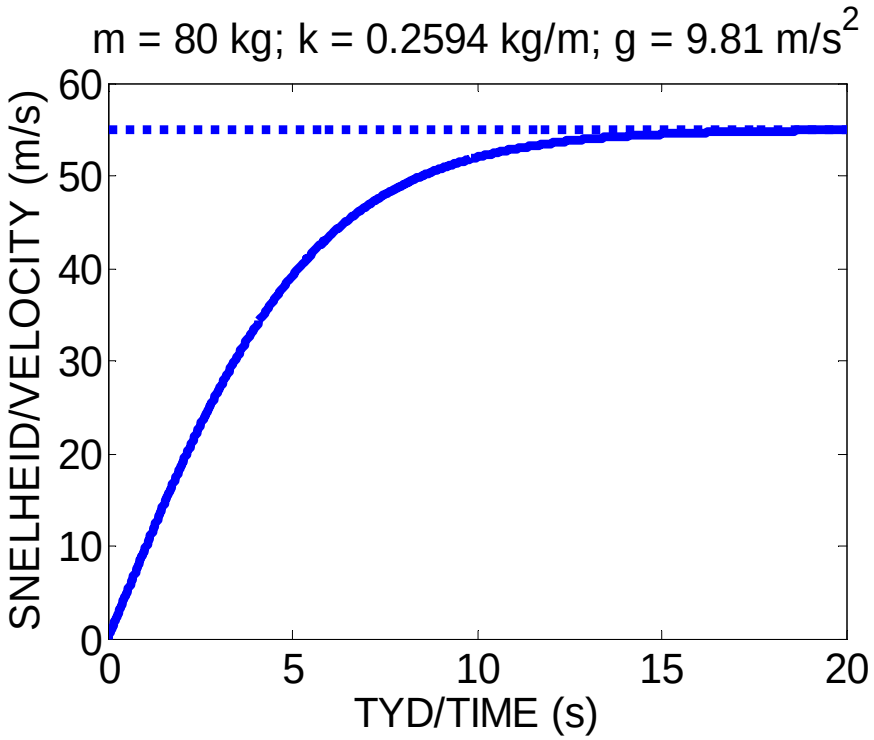
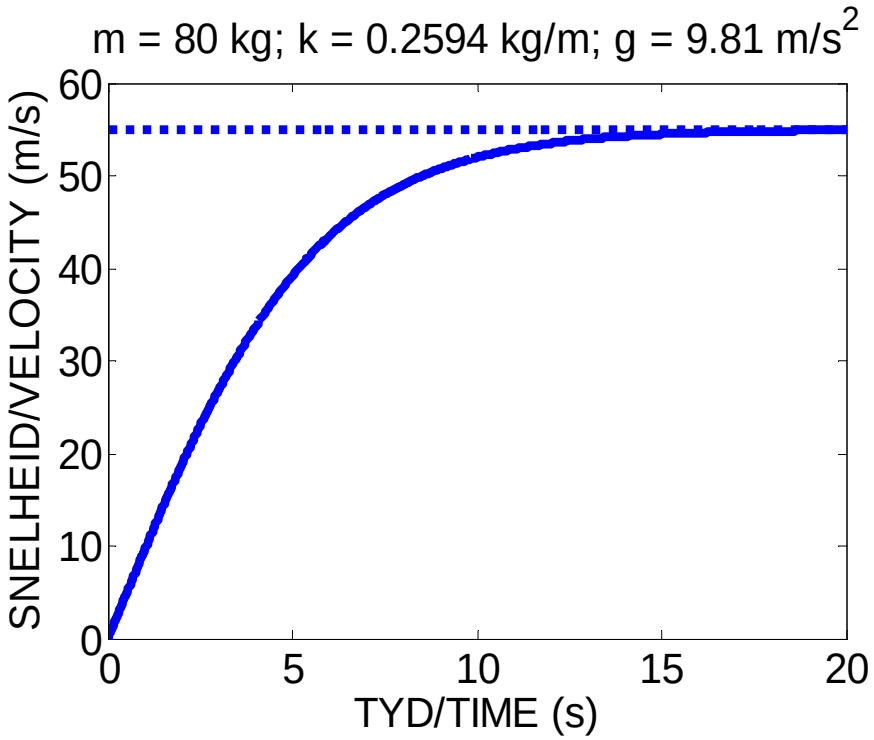
Vermenigvuldig met $e^{\sqrt{\frac{gk}{m}}t}$ bo en onder / *Multiply with $e^{\sqrt{\frac{gk}{m}}t}$ above and below:*

$$v(t) = a\left(\frac{e^{\sqrt{\frac{gk}{m}}t} - e^{-\sqrt{\frac{gk}{m}}t}}{e^{\sqrt{\frac{gk}{m}}t} + e^{-\sqrt{\frac{gk}{m}}t}}\right) = \sqrt{\frac{mg}{k}}\tanh\left(\sqrt{\frac{gk}{m}} \cdot t\right)$$


m = 80 kg; k = 0.2594 kg/m; g = 9.81 m/s2
SNELHEID/VELOCITY (m/s)
TYD/TIME (s)
0
10
20
30
40
50
60
0
5
10
15
20



Voorbeeld: 'n Man in 'n valskerm val vanuit rus en bereik 'n limietsnelheid van 4.9 m/s. Hoe lank sal dit neem om 2.45 m/s te bereik? Aanvaar dat lugweerstand direk eweredig is aan die kwadraat van die snelheid en neem $g = 9.8$ m/s^2. *Example: A man in a parachute falls from rest and reaches a limit velocity of 4.9 m/s. How long will it take him to reach 2.45 m/s? Assume that air resistance is directly proportional to the velocity squared and take $g = 9.8$ m/s^2.*



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Antwoord / *Answer:* 0.2746 **sekondes / *seconds***