1.3: App 7: Free-fall against air resistance

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<u>Without air resistance</u>

Let downward direction be positive

Displacement $y(t)$
Velocity: $v(t)$

Possible scenario: Person falls from airplane...

$\Rightarrow$ From rest: $y(0) = y_0 = 0$
$v(0) = v_0 = 0$

Newton's second law:

Resulting force = mass × acceleration ($F = ma$)

$$\text{Therefore:} \quad mg = m\frac{dv}{dt} \;\Rightarrow\; \frac{dv}{dt} = g$$

$$\Rightarrow\; v = gt + v_0 \;\Rightarrow\; \boxed{v(t) = gt} \;\Rightarrow\; \frac{dy}{dt} = gt$$

$$\Rightarrow\; y = \frac{1}{2}gt^2 + y_0 \;\Rightarrow\; \boxed{y(t) = \frac{1}{2}gt^2}$$

Free-fall against air resistance Tr 2

With air resistance

Assume: air resistance $\propto$ velocity

Newton's second law:

Resulting force = mass $\times$ acceleration $(F = ma)$

Therefore: $m\dfrac{dv}{dt} = mg - kv$ with $v(0) = v_0 = 0$

$$\text{Standard form:} \quad \frac{dv}{dt} + \frac{k}{m}v = g$$

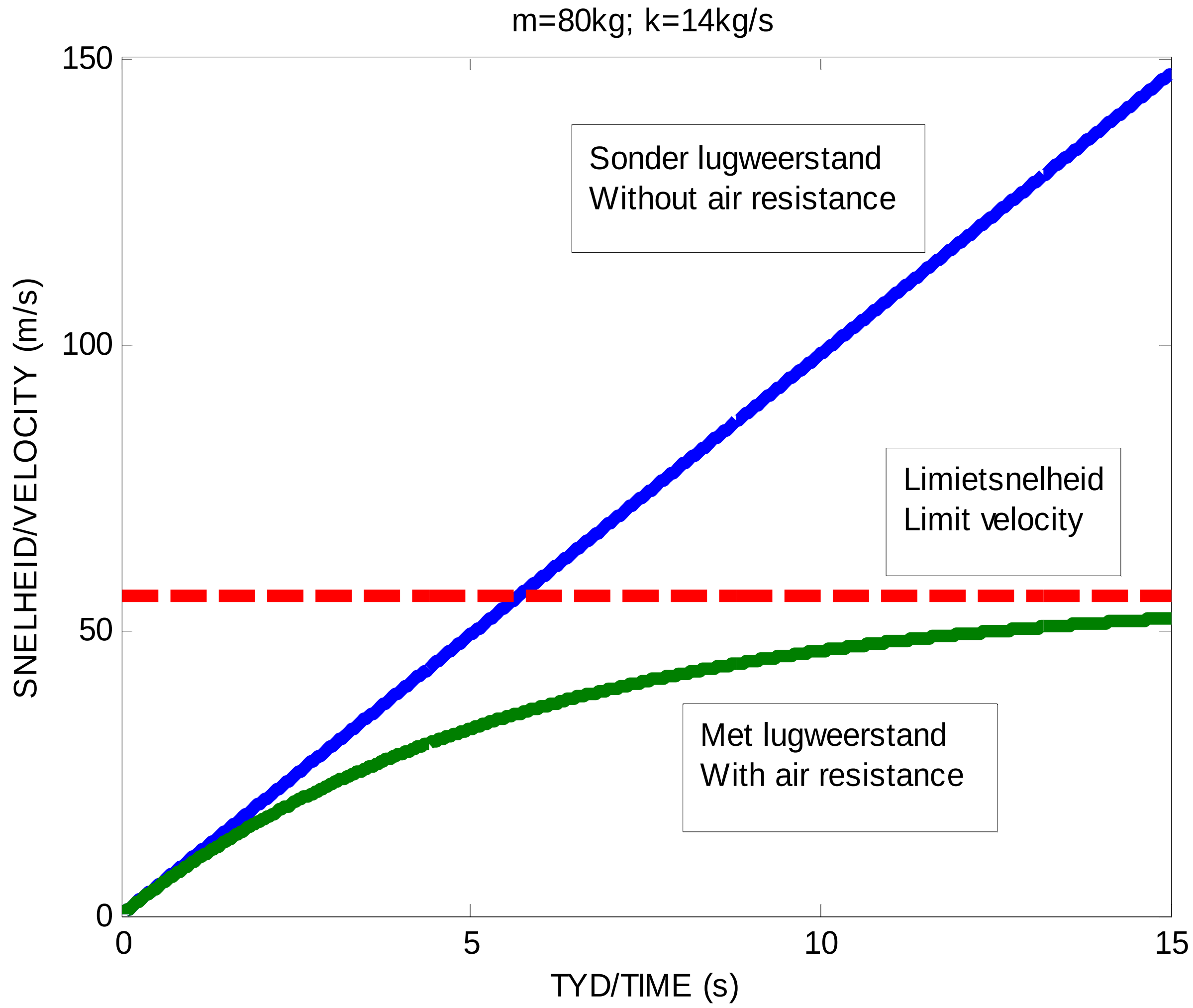
$$\text{Integration factor:} \quad I = e^{kt/m}$$

$$\text{First show that:} \quad \boxed{v(t) = \frac{mg}{k}\left(1 - e^{-kt/m}\right)}$$

$$\text{Limit velocity:} \quad \frac{mg}{k} \quad \text{(verify unit)}$$

Then show that: $\boxed{y(t) = \dfrac{mg}{k}\left(t + \dfrac{m}{k}e^{-kt/m} - \dfrac{m}{k}\right)}$

Free-fall against air resistance Tr 3



Example: A man in a parachute falls from rest and reaches a limit velocity of 4.9 m/s. How long will it take him to reach 2.45 m/s? Assume that air resistance is directly proportional to the velocity and take $g = 9.8$ m/s^2.

<u>Answer:</u> 0.3466 seconds