3.9 Eiewaardes en eiefunksies

(bladsy 167)

Probleem: Gegee die DV,

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad 0 < x < L,$$

met randvoorwaardes $y(0) = 0$ **en** $y(L) = 0$**, bestaan daar konstantes** λ **waarvoor die DV 'n nie-triviale oplossing het?**

NB: Die triviale oplossing is $y \equiv 0$

3.9 Eigenvalues and eigenfunctions

(page 167)

Problem: Given the DE,

with boundary conditions $y(0) = 0$ **and** $y(L) = 0$**, do constants** λ **exist for which the DE has a non-trivial solution?**

NB: The trivial solution is $y \equiv 0$

Geval I/*Case I:* $\boxed{\lambda = 0}$

$$\frac{d^2y}{dx^2} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = c_1 \quad \Rightarrow \quad y = c_1x + c_2$$

$$y(0) = 0 \quad \Rightarrow \quad c_2 = 0; \qquad y(L) = 0 \quad \Rightarrow \quad c_1 = 0$$

$\Rightarrow \quad y \equiv 0 \quad \Rightarrow$ **triviale oplossing!**/*trivial solution!*

$\Rightarrow \quad \lambda = 0$ **werk nie!**/*does not work!*

Geval II/*Case II:* $\boxed{\lambda < 0}$

$$\frac{d^2y}{dx^2} = \overbrace{-\lambda}^{\boxed{>0}} y$$

$$y(x) = c_1 \cosh(\sqrt{-\lambda}\, x) + c_2 \sinh(\sqrt{-\lambda}\, x)$$

$$y(0) = 0 \quad \Rightarrow \quad c_1 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad c_2 = 0$$

$\Rightarrow \quad y \equiv 0 \quad \Rightarrow$ **triviale oplossing!**/*trivial solution!*

$\Rightarrow \quad \lambda < 0$ **werk nie!**/*does not work!*

Geval III/*Case III:* $\boxed{\lambda > 0}$

$$\frac{d^2y}{dx^2} = \overbrace{-\lambda}^{\boxed{<0}} y$$

$$y(x) = c_1 \cos(\sqrt{\lambda}\, x) + c_2 \sin(\sqrt{\lambda}\, x)$$

$$y(0) = 0 \quad \Rightarrow \quad c_1 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad 0 = c_2 \sin(\sqrt{\lambda}\, L)$$

Nou kan $\sin(\sqrt{\lambda}\, L)$ egter nul wees! *Now $\sin(\sqrt{\lambda}\, L)$ can be zero however!*

$$\Rightarrow \quad \sqrt{\lambda}\, L = n\pi, \quad n = 1, 2, 3, \ldots$$

$$\Rightarrow \quad \lambda_n = \frac{n^2\pi^2}{L^2}, \quad n = 1, 2, 3, \ldots$$

$$\Rightarrow \quad y_n = c_2 \sin\left(\frac{n\pi}{L} x\right), \quad n = 1, 2, 3, \ldots$$

Let wel: $n \neq 0$ (vir $n = 0$ word die triviale oplossing verkry!)

Note: $n \neq 0$ (for $n = 0$, the trivial solution is obtained!)

Opsommend: Die differensiaalvergelyking / *In summary: The differential eqn*

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad 0 < x < L, \quad y(0) = 0, \quad y(L) = 0$$

het nie-triviale oplossings slegs wanneer $\lambda = \lambda_n$, **waar** / *has non-trivial solutions only when* $\lambda = \lambda_n$, *where*

$$\lambda_n = \frac{n^2\pi^2}{L^2}, \ n = 1, 2, 3, \ldots$$ (**Eiewaardes**/*Eigenvalues*)

$$y_n = C \sin\left(\frac{n\pi}{L} x\right), \ n = 1, 2, 3, \ldots$$ (**Eiefunksies**/*Eigenfunctions*)

Die konstante C **kan enige waarde aanneem, solank** $C \neq 0$

The constant C *can take any value, as long as* $C \neq 0$

Neem $C = 1$ **en skets** y_1, y_2 **en** y_3 / *Take* $C = 1$ *and sketch* y_1, y_2, *and* y_3