3.9 Eiewaardes en eiefunksies *(bl 167)*

Probleem: Gegee die DV,

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad 0 < x < L,$$

met randvoorwaardes $y(0) = 0$ en $y(L) = 0$, bestaan daar konstantes λ waarvoor die DV 'n nie-triviale oplossing het?

Let wel: Die triviale oplossing is $y \equiv 0$

Geval I: $\boxed{\lambda = 0}$

$$\frac{d^2y}{dx^2} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = c_1 \quad \Rightarrow \quad y = c_1x + c_2$$

$$y(0) = 0 \quad \Rightarrow \quad c_2 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad c_1 = 0$$

$$\Rightarrow \quad y \equiv 0 \quad \Rightarrow \quad \text{triviale oplossing!}$$

$$\Rightarrow \quad \lambda = 0 \quad \text{werk nie!}$$

Geval II: $\boxed{\lambda < 0}$

$$\frac{d^2y}{dx^2} = \overbrace{-\lambda}^{>0}\, y$$

$$y(x) = c_1 \cosh(\sqrt{-\lambda}\, x) + c_2 \sinh(\sqrt{-\lambda}\, x)$$

$$y(0) = 0 \quad \Rightarrow \quad c_1 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad c_2 = 0$$

$\Rightarrow \quad y \equiv 0 \quad \Rightarrow \quad$ triviale oplossing!

$\Rightarrow \quad \lambda < 0$ werk nie!

Geval III: $\boxed{\lambda > 0}$

$$\frac{d^2y}{dx^2} = \overbrace{-\lambda}^{<0}\, y$$

$$y(x) = c_1 \cos(\sqrt{\lambda}\, x) + c_2 \sin(\sqrt{\lambda}\, x)$$

$$y(0) = 0 \quad \Rightarrow \quad c_1 = 0$$

$$y(L) = 0 \quad \Rightarrow \quad 0 = c_2 \sin(\sqrt{\lambda}\, L)$$

Nou kan $\sin(\sqrt{\lambda}\, L)$ egter nul wees!

$$\Rightarrow \quad \sqrt{\lambda}\, L = n\pi, \quad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \quad \lambda_n = \frac{n^2\pi^2}{L^2}, \quad n = 0, 1, 2, 3, \ldots$$

$$\Rightarrow \quad y_n = c_2 \sin\left(\frac{n\pi}{L} x\right), \quad n = 0, 1, 2, 3, \ldots$$

Let wel: $n \neq 0$ (vir $n = 0$ word die triviale oplossing verkry!)

Opsommend: Die DV

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad 0 < x < L, \quad y(0) = 0, \quad y(L) = 0$$

het nie-triviale opl's slegs wanneer $\lambda = \lambda_n$, waar

Eiewaardes en eiefunksies Tr 4

$$\lambda_n = \frac{n^2\pi^2}{L^2},\ n = 1, 2, 3, \ldots \ \textbf{(Eiewaardes)}$$

$$y_n = C\,\sin\big(\frac{n\pi}{L}\,x\big),\ n = 1, 2, 3, \ldots \ \textbf{(Eiefunksies)}$$

Die konstante C kan enige waarde aanneem, solank $C \neq 0$

Neem $C = 1$ en skets y_1, y_2 en y_3