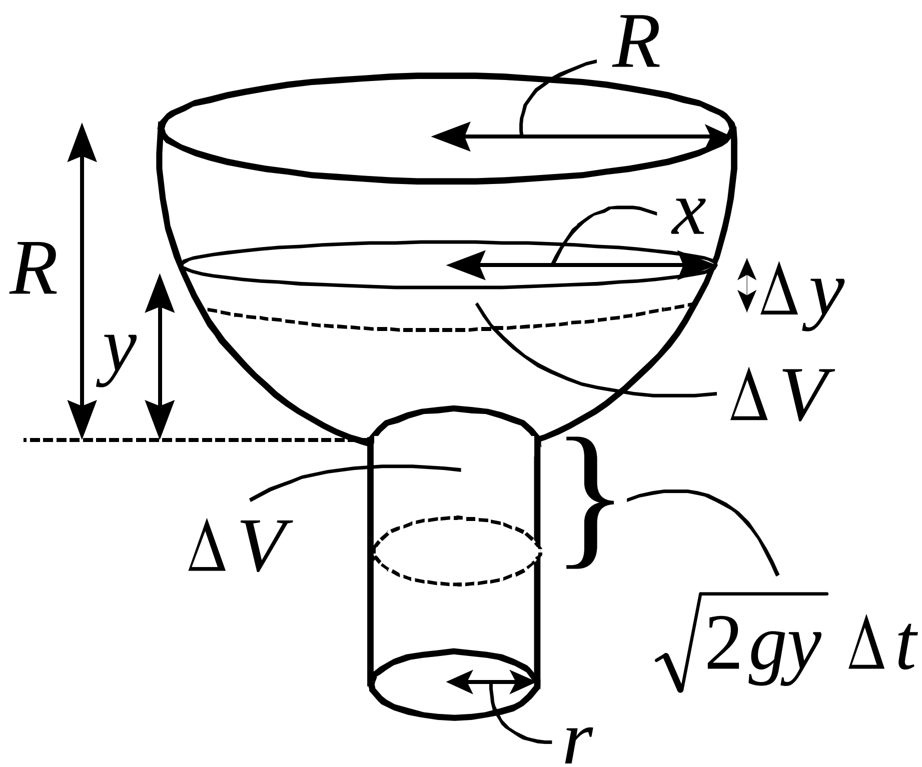
Vloei van water uit 'n tenk
(Hemisferiese tenk)

Flow of water from a tank
(Hemispherical tank)

Voorbeeld (waar A_w nie konstant is nie): Die hemisferiese tenk met radius R is aanvanklik heeltemal gevul met water. Ignoreer wrywing/samepersing van water by die gat (met radius r) en aanvaar dat Toricelli se wet geld. Hoe lank sal dit neem vir al die water om uit te loop?

Example (where A_w is not constant): The hemispherical tank with radius R is initially entirely filled with water. Ignore friction/contraction of water at the hole (with radius r) and assume that Toricelli's law applies. How long will it take for all the water to run out?



Laat $y = y(t)$ die hoogte van die watervlak in die tenk wees
Let $y = y(t)$ be the height of the water level in the tank

$$\Delta V = -(\sqrt{2gy} \cdot \Delta t)(\pi r^2)$$

maar / *but* $\Delta V = \pi x^2 \cdot \Delta y$

en / *and* $(R - y)^2 + x^2 = R^2$

$$\Rightarrow \quad x^2 = R^2 - (R^2 - 2yR + y^2) = 2yR - y^2$$

dus / *therefore* $\Delta V = \pi(2yR - y^2) \cdot \Delta y = -(\sqrt{2gy} \cdot \Delta t)(\pi r^2)$

$$\Rightarrow \quad (2yR - y^2)\frac{\Delta y}{\Delta t} = -\sqrt{2gy} \cdot r^2$$

As / *If* $\Delta t \to 0$ **dan** / *then* $\dfrac{\Delta y}{\Delta t} \to \dfrac{dy}{dt}$

$\Rightarrow$ **DV** / *DE:* $\boxed{(2yR - y^2)\dfrac{dy}{dt} = -\sqrt{2gy} \cdot r^2, \; y(0) = R}$

Oplossing: Integrasiefaktore gaan nie werk nie, want DV is nie-lineêr

Solution: Integration factors will not work, since DE is non-linear

Gebruik dus skeiding van veranderlikes en toon aan dat:

Therefore use separation of variables and show that:

$$\boxed{\frac{4}{3}y^{3/2}R - \frac{2}{5}y^{5/2} = -r^2\sqrt{2g}\cdot t + \frac{14}{15}R^{5/2}}$$

$\Rightarrow$ **Implisiete oplossing:** y **kan nie eksplisiet in terme van** t **geskryf word nie**

$\Rightarrow$ *Implicit solution:* y *can not be written explicitely in terms of* t

Die tenk is leeg wanneer $y = 0$

The tank is empty when $y = 0$

$$\Rightarrow \quad t = \frac{\frac{14}{15}R^{5/2}}{r^2\sqrt{2g}}$$