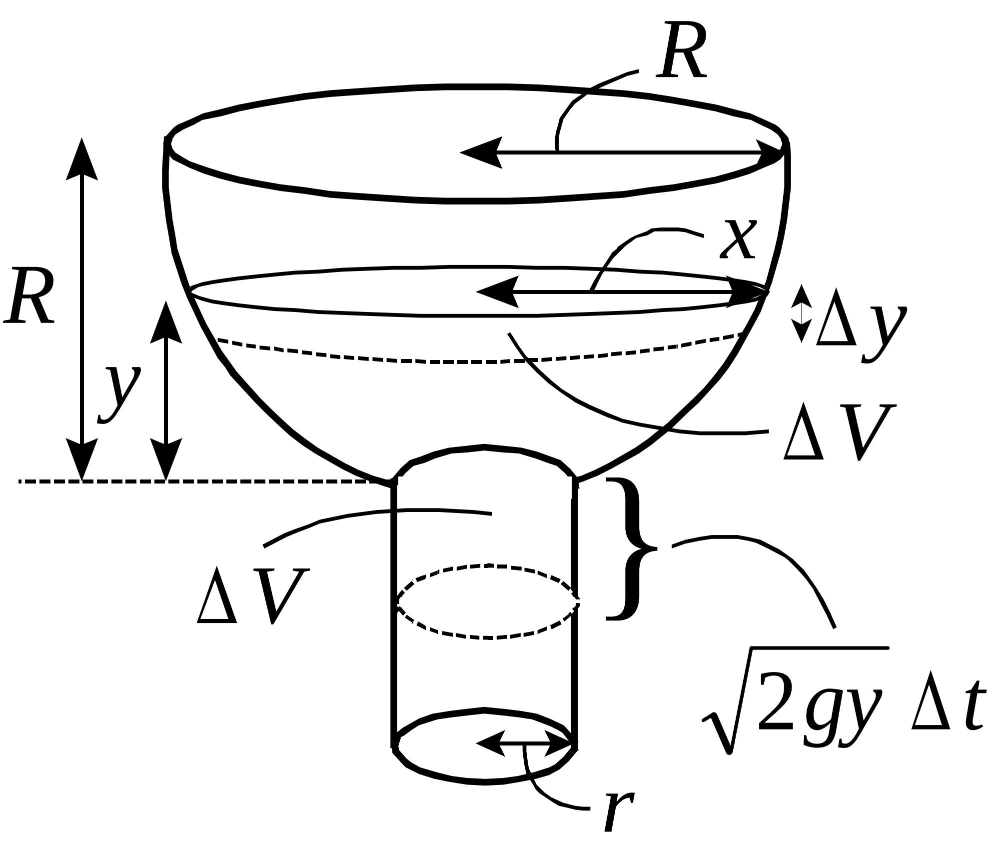
Application: Flow of water from a tank

Example (where A_w is not constant): The hemispherical tank with radius R is initially entirely filled with water. Ignore friction/contraction of water at the hole (with radius r) and assume that Toricelli's law applies. How long will it take for all the water to run out?



Drainage of a hemispherical tank — Tr 2

Let $y = y(t)$ be the height of the water level in the tank

$$\Delta V = -(\sqrt{2gy} \cdot \Delta t)(\pi r^2)$$

$$\text{but} \quad \Delta V = \pi x^2 \cdot \Delta y$$

$$\text{and} \quad (R - y)^2 + x^2 = R^2$$

$$\Rightarrow \quad x^2 = R^2 - (R^2 - 2yR + y^2) = 2yR - y^2$$

$$\text{thus} \quad \Delta V = \pi(2yR - y^2) \cdot \Delta y = -(\sqrt{2gy} \cdot \Delta t)(\pi r^2)$$

$$\Rightarrow \quad (2yR - y^2)\frac{\Delta y}{\Delta t} = -\sqrt{2gy} \cdot r^2$$

If $\Delta t \to 0$ then $\dfrac{\Delta y}{\Delta t} \to \dfrac{dy}{dt}$

$$\Rightarrow \text{ DE: } \boxed{(2yR - y^2)\frac{dy}{dt} = -\sqrt{2gy} \cdot r^2, \; y(0) = R}$$

Drainage of a hemispherical tank Tr 3

Solution: Integration factors will not work, since DE is non-linear

Therefore use separation of variables and show that:

$$\boxed{\frac{4}{3}y^{3/2}R - \frac{2}{5}y^{5/2} = -r^2\sqrt{2g}\cdot t + \frac{14}{15}R^{5/2}}$$

$\Rightarrow$ Implicit solution: y can not be written explicitely in terms of t

The tank is empty when $y = 0$

$$\Rightarrow \quad t = \frac{\frac{14}{15}R^{5/2}}{r^2\sqrt{2g}}$$