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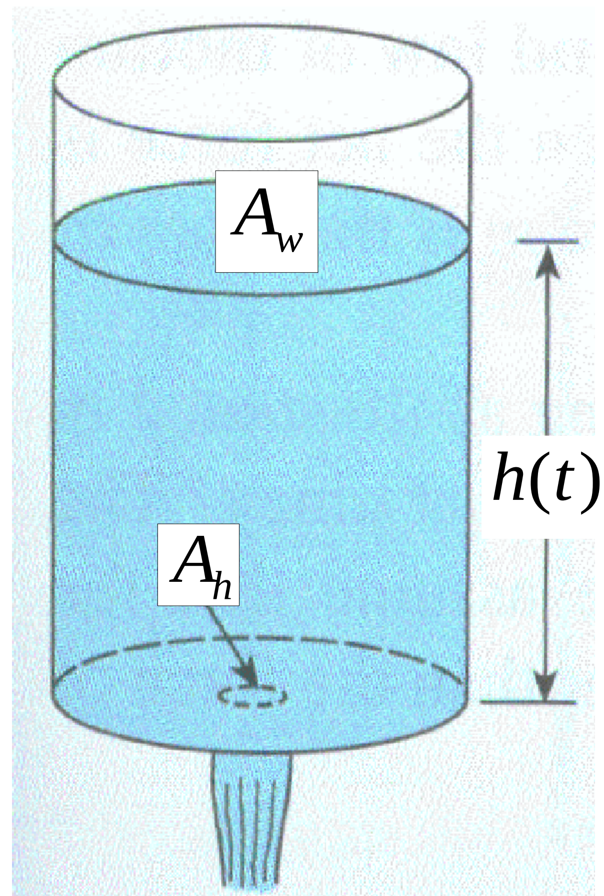

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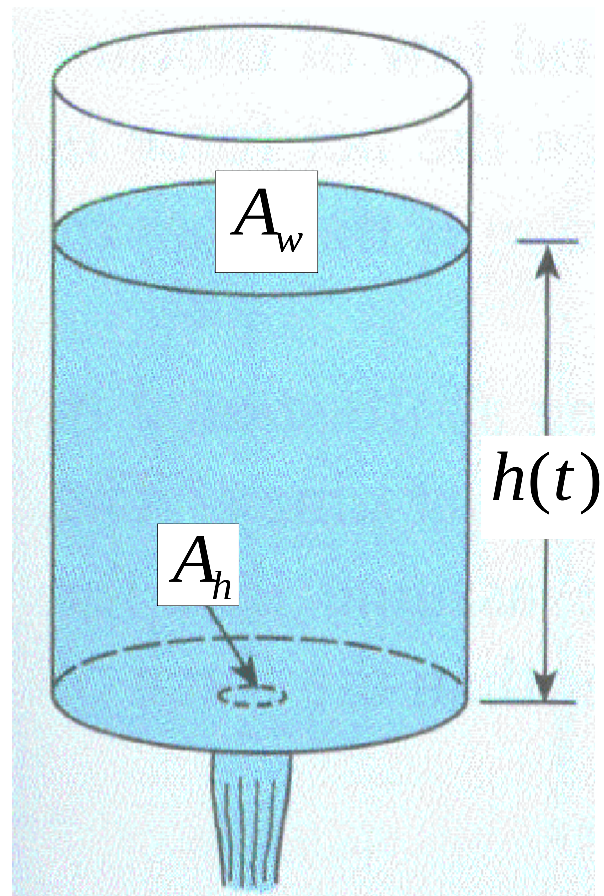
Laat $h = h(t)$ die hoogte van die watervlak op tyd t wees, A_w die oppervlak-area van die watervlak, A_h die oppervlak-area van die gaatjie, en g swaartekragversnelling. *Let $h = h(t)$ be the height of the water level at time t, A_w the surface area of the water level, A_h the surface area of the hole, and g gravitational acceleration.*



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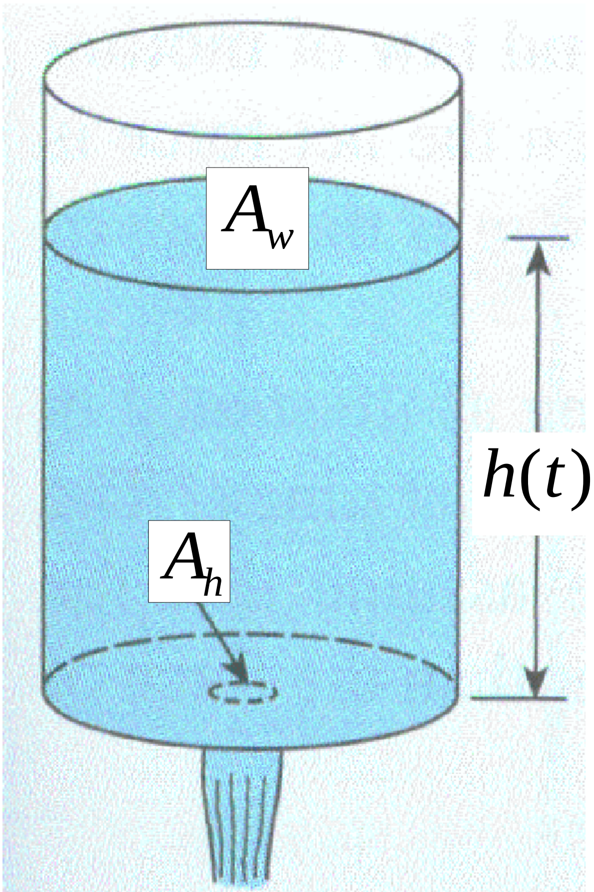
Formuleer: Laat $V = V(t)$ die volume water in die tenk op tystip t wees

Formulate: Let $V = V(t)$ be the volume of the water in the tank at time t



Watervlak daal Δh in tyd Δt

Water level drops Δh in time Δt

$$\Delta V = -\left(\sqrt{2gh}\cdot\Delta t\right) A_h$$

$$\frac{\Delta V}{\Delta t} = \frac{dV}{dt} = -A_h\sqrt{2gh}$$

$$V = A_w h$$

$$\frac{dh}{dt} = -\frac{A_h}{A_w}\sqrt{2g}\,h^{1/2}$$

$$\frac{dh}{dt} = -\frac{A_h}{A_w}\sqrt{2g}\,h^{1/2} = -\rho\sqrt{2g}\,h^{1/2}, \quad h(0) = h_0$$

Oplossing: Integrasiefaktore gaan nie werk nie, want DV is nie-lineêr

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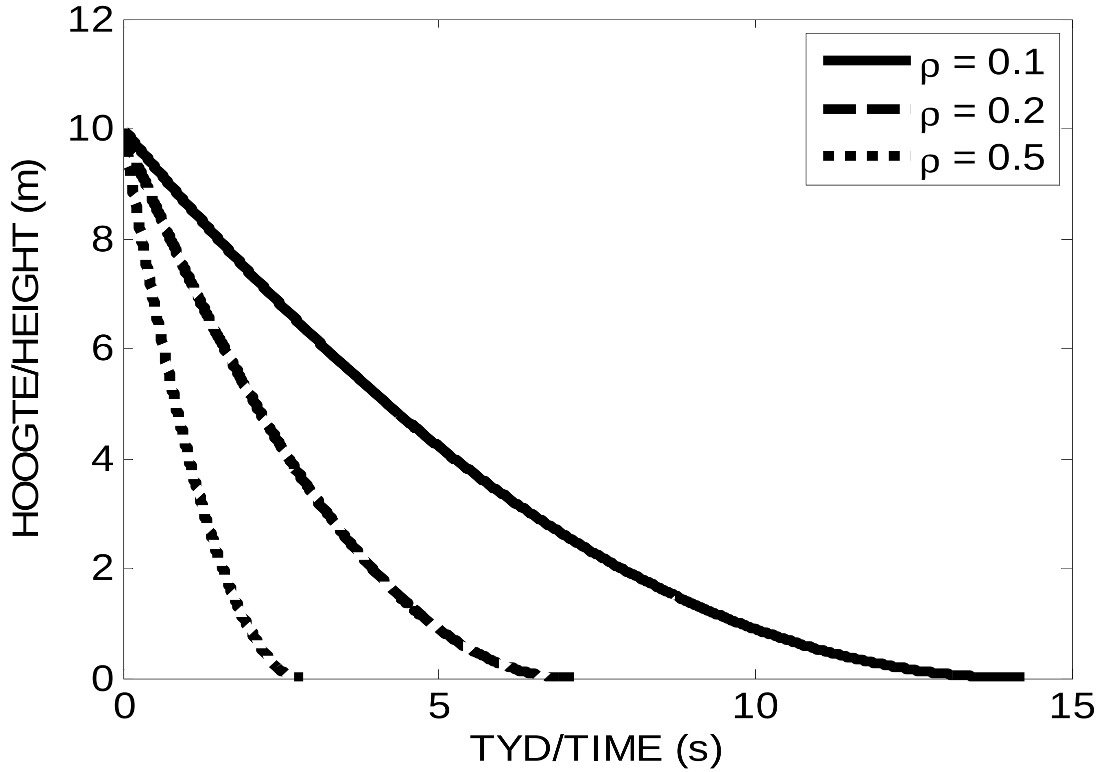
Toets: Gestel die tenk is onder heeltemal oop, dan / *Test: Suppose that the tank is entirely open at the bottom, then*

$$h_0 = \frac{1}{2} g t^2 \ \textbf{en / } \textit{and} \ \ t = \frac{{h_0}^{1/2}}{1} \sqrt{\frac{2}{g}} \Rightarrow \rho = 1$$

Voorbeeld / *Example* ($h_0 = 10$ **m;** $g = 9.81$ **m/s**^{2})



Grafiek vir algemene geval: $\dfrac{dh}{dt}$ **is altyd negatief** $\Rightarrow$ **grafiek is altyd dalend**

Graph for general case: $\dfrac{dh}{dt}$ *is always negative* $\Rightarrow$ *graph is always decreasing*

Toon dat / Show that $\dfrac{d^2h}{dt^2} = \rho^2 g > 0 \Rightarrow$ **konkaaf na bo / concave upwards**