1.3: Differensiaalvergelykings as Wiskundige modelle (bl 18)

1.3: Differential Equations as Mathematical Models (p 18)

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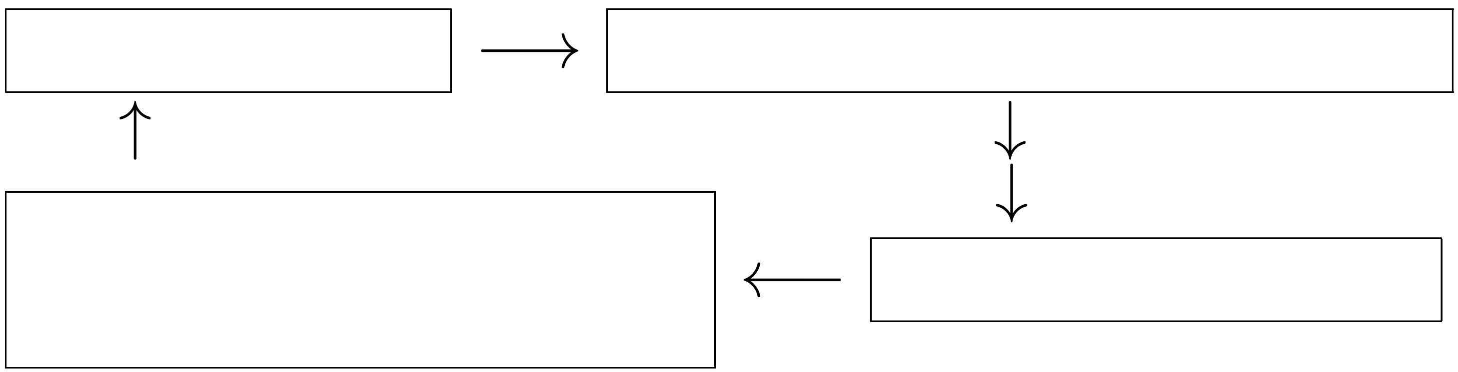
Beginsels van modellering: **(Iteratiewe proses!)**

1.3: Differential Equations as Mathematical Models (p 18)

Principles of modelling: **(Iterative process!)**

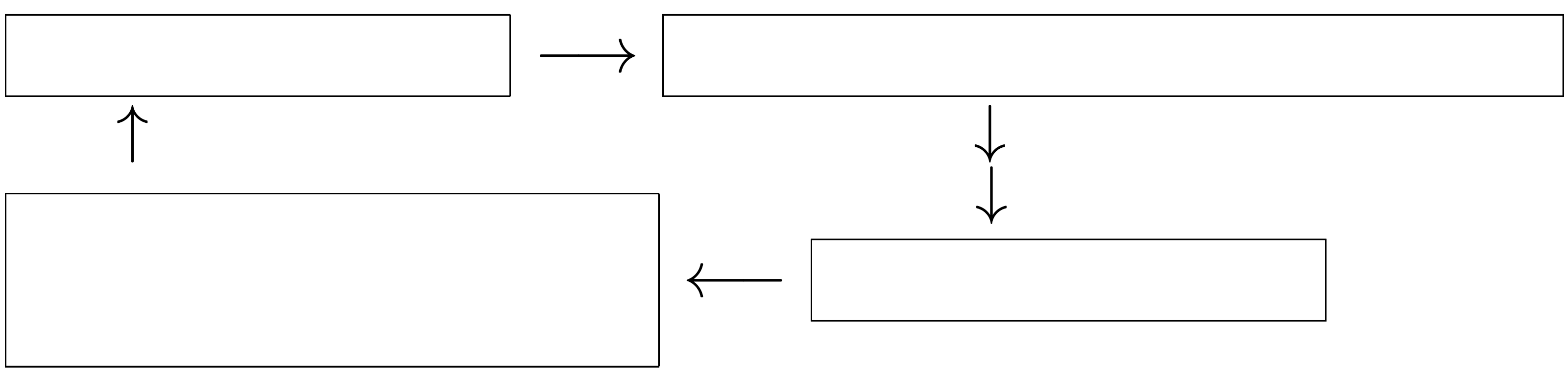
1.3: Differensiaalvergelykings as Wiskundige modelle (bl 18)

Beginsels van modellering: **(Iteratiewe proses!)**



1.3: Differential Equations as Mathematical Models (p 18)

Principles of modelling: **(Iterative process!)**



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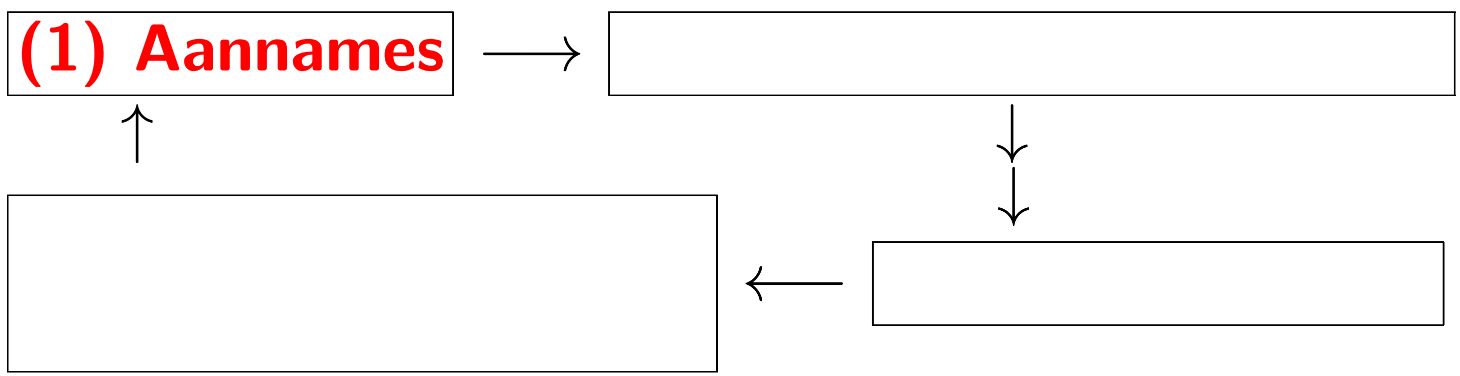
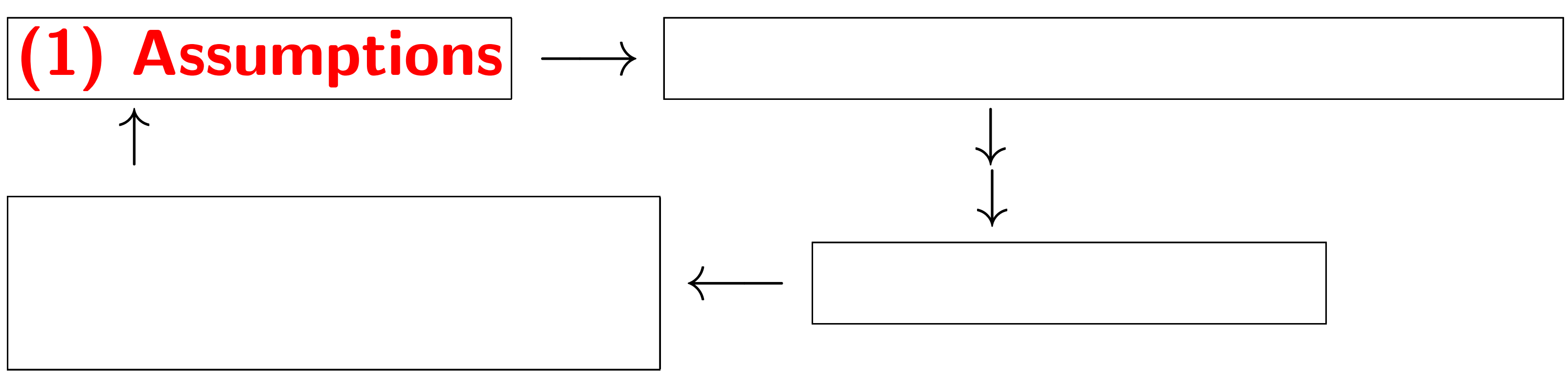
Beginsels van modellering: **(Iteratiewe proses!)**



1.3: Differential Equations as Mathematical Models (p 18)

Principles of modelling: **(Iterative process!)**



1.3: Differensiaalvergelykings as Wiskundige modelle (bl 18)

Beginsels van modellering: **(Iteratiewe proses!)**

(1) Aannames → (2) Wiskundige formulering

1.3: Differential Equations as Mathematical Models (p 18)

Principles of modelling: **(Iterative process!)**

(1) Assumptions → (2) Mathematical formulation

1.3: Differensiaalvergelykings as Wiskundige modelle (bl 18)

Beginsels van modellering: **(Iteratiewe proses!)**

(1) Aannames ⟶ (2) Wiskundige formulering
↓
(3) Vind oplossing ⟶ [ ] ⟶ (1)

1.3: Differential Equations as Mathematical Models (p 18)

Principles of modelling: **(Iterative process!)**

(1) Assumptions ⟶ (2) Mathematical formulation
↓
(3) Find solution ⟶ [ ] ⟶ (1)

1.3: Differensiaalvergelykings as Wiskundige modelle (bl 18)

Beginsels van modellering: **(Iteratiewe proses!)**

(1) Aannames ⟶ (2) Wiskundige formulering

↓

(3) Vind oplossing ⟶ (4) Toets voorspelling aan gemete data

↑ (4) ⟶ (1)

Stophorlosie, liniaal, ens. ⟶ (4)

1.3: Differential Equations as Mathematical Models (p 18)

Principles of modelling: **(Iterative process!)**

(1) Assumptions ⟶ (2) Mathematical formulation

↓

(3) Find solution ⟶ (4) Check prediction with measured data

↑ (4) ⟶ (1)

Stopwatch, ruler, etc. ⟶ (4)

Illustrasie: Beskou ’n projektiel wat vertikaal beweeg. Wat is die verplasing y op enige tydstip t?

(1) Aannames:
- Beweging reglynig
- Lugweerstand geïgnoreer
- Gravitasieversnelling konstant

Illustration: Consider a projectile that moves vertically. What is the displacement y at any given time t?

(1) Assumptions:
- Motion linear
- Air resistance ignored
- Gravitational acceleration constant

Illustrasie: Beskou 'n projektiel wat vertikaal beweeg. Wat is die verplasing y op enige tydstip t?

(2) Wiskundige formulering:

- Laat $y = y(t)$ die verplasing op enige tydstip t wees (opwaarts positief)

Illustration: Consider a projectile that moves vertically. What is the displacement y at given time t?

(2) Mathematical formulation:

- Let $y = y(t)$ be the displacement at any given time t (positive upwards)

(2) Wiskundige formulering...

Newton se 2de wet: Resulterende krag $=$ **massa** $\times$ **versnelling** **(**$F = ma$**)**

$$\textbf{Dus:} \quad -mg = m\frac{d^2y}{dt^2} \quad \Rightarrow \quad \frac{d^2y}{dt^2} = -g$$

---

(2) Mathematical formulation...

Newton's 2nd law: Resultant force $=$ **mass** $\times$ **acceleration** **(**$F = ma$**)**

$$\textbf{Therefore:} \quad -mg = m\frac{d^2y}{dt^2} \quad \Rightarrow \quad \frac{d^2y}{dt^2} = -g$$

(3) Verkry oplossings: Differensiaalvergelyking: $\dfrac{d^2y}{dt^2} = -g$

Integreer: $\dfrac{dy}{dt} = -gt + C$

Gestel $\dfrac{dy}{dt} = u$ **(aanvangsnelheid) as** $t = 0$

Dan $C = u$ **sodat** $\dfrac{dy}{dt} = u - gt$

---

(3) Obtain solutions: Differential equation: $\dfrac{d^2y}{dt^2} = -g$

Integrate: $\dfrac{dy}{dt} = -gt + C$

Suppose $\dfrac{dy}{dt} = u$ **(initial velocity) when** $t = 0$

Then $C = u$ **so that** $\dfrac{dy}{dt} = u - gt$

(3) Verkry oplossings... $$\frac{dy}{dt} = u - gt$$

Integreer (weer): $$y = ut - \frac{1}{2}gt^2 + K$$

Gestel $y = 0$ **as** $t = 0$, **dan** $K = 0$ **sodat** $\boxed{y = ut - \frac{1}{2}gt^2}$

---

(3) Obtain solutions... $$\frac{dy}{dt} = u - gt$$

Integrate (again): $$y = ut - \frac{1}{2}gt^2 + K$$

Suppose $y = 0$ **when** $t = 0$, **then** $K = 0$ **so that** $\boxed{y = ut - \frac{1}{2}gt^2}$

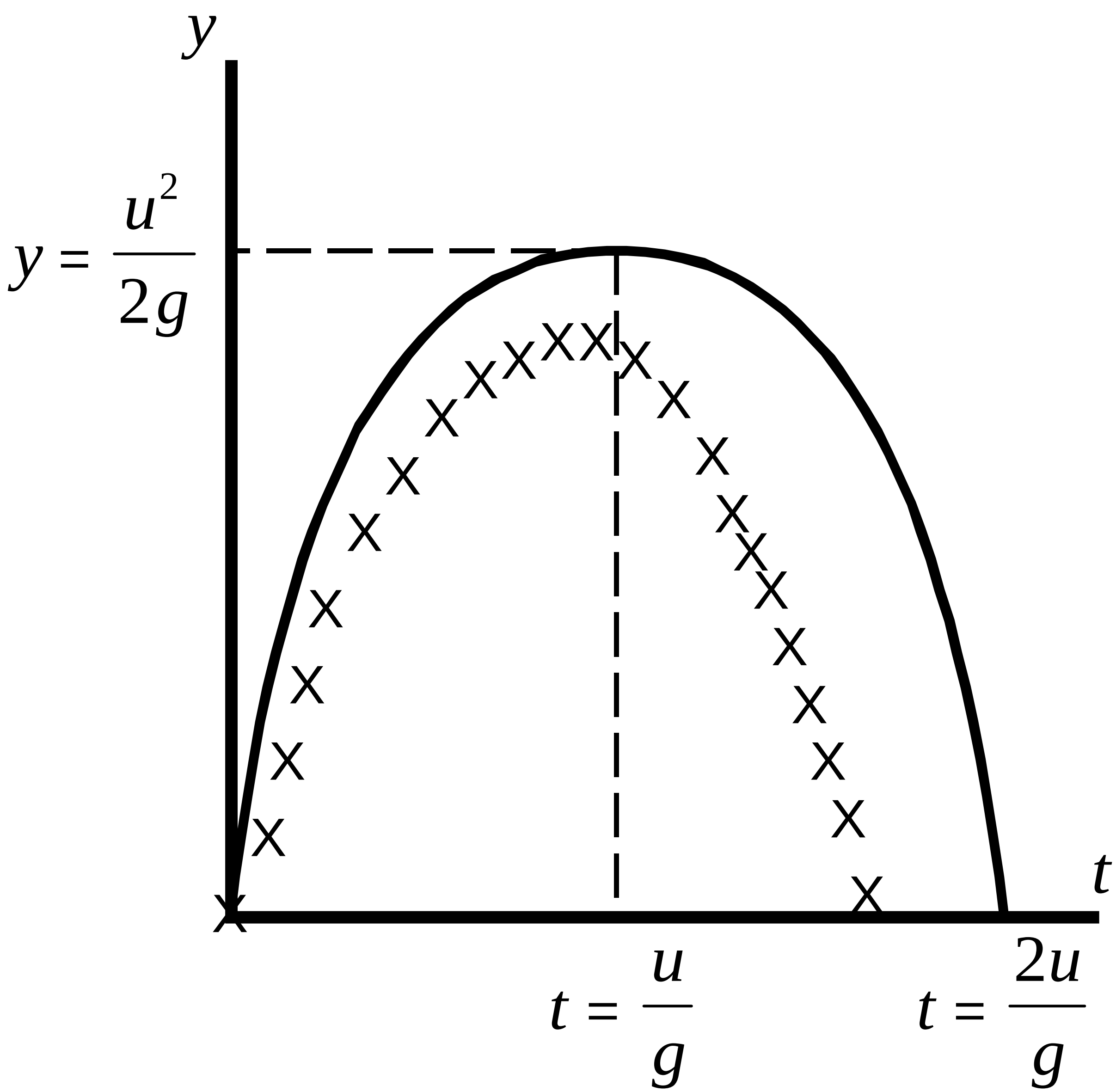
(4) Toets voorspelling aan gemete data:

Bekende (gemete) data ($\times$) word dan geplot teenoor die wiskundige model se voorspelling ($-$) ...

---

(4) Check prediction with measured data:

Known (measured) data ($\times$) is then plotted against the mathematical model's prediction ($-$) ...


y
$y = \frac{u^2}{2g}$
t
$t = \frac{u}{g}$
$t = \frac{2u}{g}$

Die aannames moet dus verander (verbeter) word om 'n meer akkurate model te verkry!

Aanvaar bv dat: lugweerstand $\propto$ snelheid dws $R = r\dfrac{dy}{dt}$, waar r 'n konstante is... $\Rightarrow$ [$F = ma$] $-mg - r\dfrac{dy}{dt} = m\dfrac{d^2y}{dt^2}$

(Bogenoemde slegs geldig vir opwaartse beweging!)

Los op en herhaal die proses... (LATER)

The assumptions therefore have to be changed (improved) in order to obtain a more accurate model!

Assume, eg, that: air resistance $\propto$ velocity, ie $R = r\dfrac{dy}{dt}$, where r is a constant... $\Rightarrow$ [$F = ma$] $-mg - r\dfrac{dy}{dt} = m\dfrac{d^2y}{dt^2}$

(The above only valid for upwards motion)

Solve and repeat the process... (LATER)