1.3: DEs as mathematical models *(Page 18)*

Principles of modelling: *(Iterative process!)*

(1) Assumptions ⟶ **(2) Math formulation**

↑ ↓

(4) Check prediction with known data ⟵ **(3) Find solution**

Illustration: Consider a projectile that moves vertically. What is the displacement y at given time t?

(1) Assumptions:
- Motion linear
- Air resistance ignored
- Gravitational acc constant

(2) Math formulation:
- Let $y = y(t)$ be the displacement at a given time t (positive upwards)

Newton's 2nd law:

Resultant force = mass × acceleration ($F = ma$)

$$\text{Therefore:} \quad -mg = m\frac{d^2y}{dt^2}$$

$$\frac{d^2y}{dt^2} = -g$$

(3) Find solutions

$$\text{Integrate:} \quad \frac{dy}{dt} = -gt + C$$

Suppose $\frac{dy}{dt} = u$ (initial velocity) when $t = 0$

Then $C = u$ so that $\frac{dy}{dt} = u - gt$

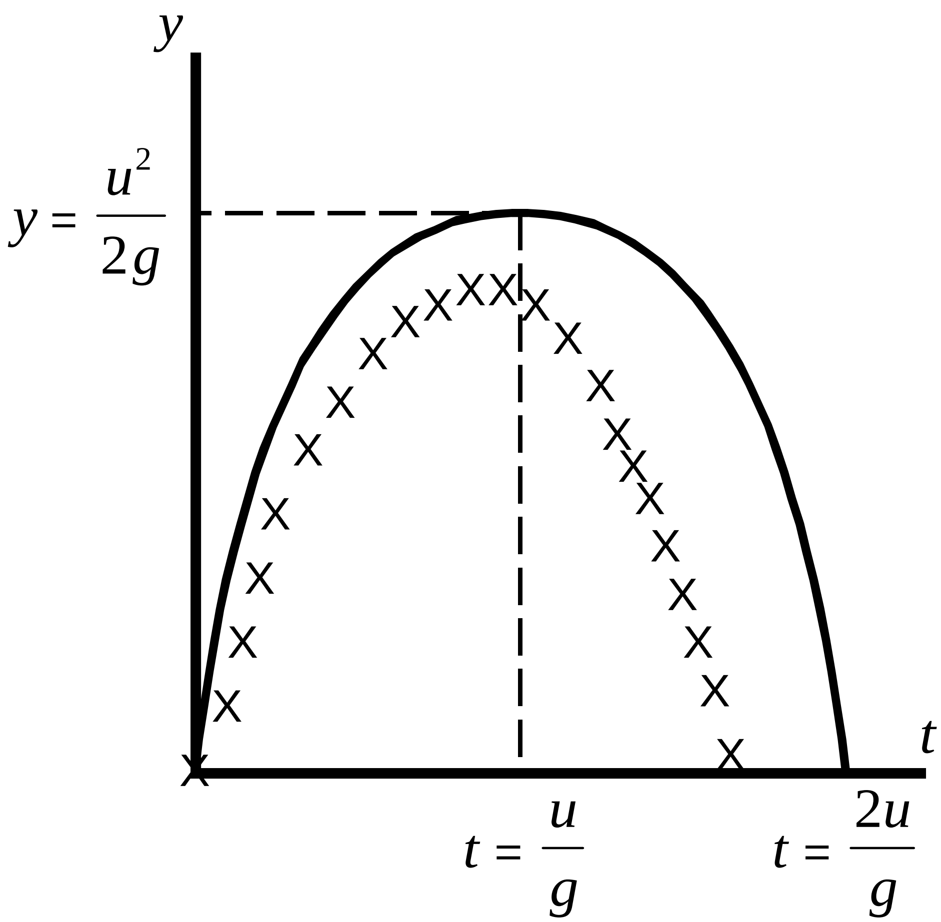
$$\text{Integrate (again):} \quad y = ut - \frac{1}{2}gt^2 + K$$

Suppose $y = 0$ when $t = 0$

Then $K = 0$ so that $\boxed{y = ut - \frac{1}{2}gt^2}$

(4) Check prediction with known data

Know (calculated) data ($\times$) is then plotted against the mathematical model's prediction ($-$) ...



The assumptions therefore have to be changed (improved) in order to obtain a more accurate model!

Assume, for example, that: air resistance $\propto$ velocity, that is $R = r\dfrac{dy}{dt}$, where r is a constant...

Newton's 2nd law:

Resultant force $=$ mass $\times$ acceleration $(F = ma)$

$$\text{Therefore:} \quad -mg - r\frac{dy}{dt} = m\frac{d^2y}{dt^2}$$

(The above only valid for upwards motion)

Solve and repeat the process... (LATER)