3.9 Uitwyking van 'n belaste balk

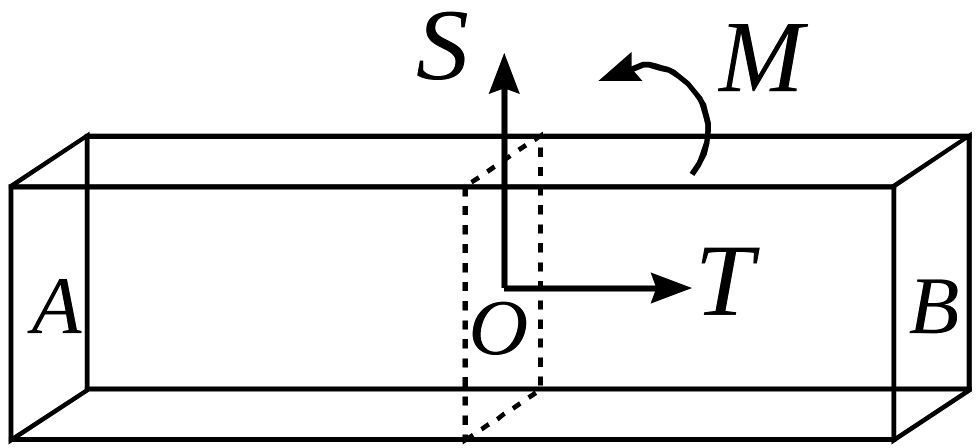
(bladsy 165)

3.9 Deflection of a loaded beam

(page 165)

Probleem: Bepaal die uitwyking van 'n balk soos in fig 3.9.2 in Z&W (sien aanvullende aantekeninge op web)

Problem: Determine the deflection of a beam as in fig 3.9.2 in Z&W (see supplementary notes on web)



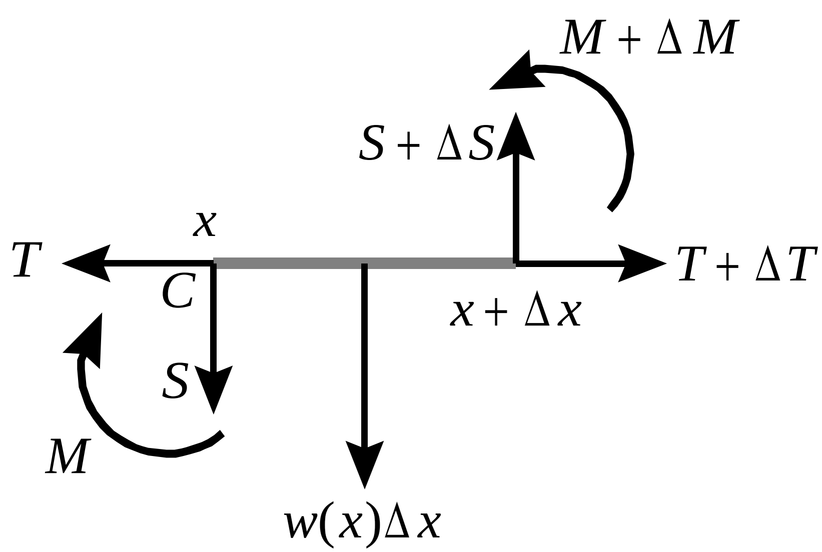
Beskou die snit O. Deel B oefen 'n trekkrag T, skuifkrag S, en buigmoment M op deel A uit. Kies x-as $\rightarrow$ en y-as $\uparrow$.

Consider a section O. Part B exerts a tension force T, shearing force S, and bending moment M onto part A. Choose x-axis $\rightarrow$ and y-axis $\uparrow$.

Neem aan: (1) Balk is belas met gewig $w(x)$ per eenheidslengte (2) Geen eksterne horisontale kragte

Assume: (1) Beam is loaded with weight $w(x)$ per unit length (2) No external horizontal forces

Beskou klein elementjie met lengte Δx / *Consider small element with length Δx:*



Vir ewewig geld / *For equilibrium we have:*

$$\boxed{(\rightarrow)\ \Sigma F_x = 0}\ T + \Delta T - T = 0 \qquad \textbf{(1)}$$

$$\boxed{(\uparrow)\ \Sigma F_y = 0}\ S + \Delta S - S - w(x)\Delta x = 0 \qquad \textbf{(2)}$$

$$\boxed{(\circlearrowleft)\ \Sigma M_C = 0}\ M + \Delta M - M + (S + \Delta S)\Delta x - (w(x)\Delta x)\tfrac{1}{2}\Delta x = 0 \qquad \textbf{(3)}$$

In vgl (3), ignoreer terme wat klein is tot 2de orde d.w.s. $\Delta S \Delta x$ en $\Delta x \Delta x$:
In eqn (3), ignore terms that are small to 2nd order i.e. $\Delta S \Delta x$ and $\Delta x \Delta x$:

$$M + \Delta M - M + S\Delta x = 0 \quad \textbf{(3')}$$

In (1), (2) en (3'), kanselleer terme en deel met Δx. Laat verder $\Delta x \to 0$ $\Rightarrow$ 3 basiese DVs van balke / *In (1), (2) and (3'), cancel terms and divide by Δx. Also let $\Delta x \to 0 \Rightarrow$ 3 basic DEs for beams:*

$$\frac{dT}{dx} = 0, \qquad \frac{dS}{dx} = w(x), \qquad \frac{dM}{dx} = -S$$

Om $y = y(x)$ te kry, differensieer die 3de van bg vgls na x, en gebruik middelste vgl om S te elimineer / *To find $y = y(x)$, differentiate the 3rd of the above eqns wrt x, and use the middle eqn to eliminate S:*

$$\frac{d^2M}{dx^2} = -\frac{dS}{dx} = -w(x) \quad \textbf{(4)}$$

Euler-Bernoulli-stelling vir dun balke: Neem aan dat $M \propto$ kromming K/
Euler-Bernoulli theorem for thin beams: Assume that $M \propto$ curvature K:

$$M = (EI)K \qquad \Rightarrow \qquad M \approx EI\,\frac{d^2y}{dx^2} \quad \textbf{(5)}$$

$E \equiv$ **"Young" modulus;** $I \equiv$ **traagheidsmoment/***moment of inertia*

Vir herleiding van $K \approx \frac{d^2y}{dx^2}$, as $\frac{dy}{dx}$ klein is (dws as uitwyking klein is): sien aanvullende aantekening (bl 2) (SELFSTUDIE)

For derivation of $K \approx \frac{d^2y}{dx^2}$, when $\frac{dy}{dx}$ is small (ie when deflection is small): see supplementary notes (page 2) (SELF STUDY)

Differensieer (5) 2 keer mbt x, en gebruik (4) om M te elimineer. Dan volg:

Differentiate (5) twice wrt x, and use (4) to eliminate M. Then follows:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) = -w(x)$$

Met Z&W se keuse van die y-as afwaarts en E en I beide konstant, volg vergelyking (4) op bl 166:

With Z&W's choice of the y-axis downwards and E and I both constant, equation (4) on page 166 follows:

$$\boxed{EI\frac{d^4y}{dx^4} = w(x)}$$

Staan bekend as BALKVERGELYKING / *Known as* **BEAM EQUATION**

Vierde orde DV: Vir unieke oplossing word vier randvoorwaardes benodig, twee links en twee regs

Forth *order DE: For unique solution* **four** *boundary conditions are required, two left and two right*

Let op: As ons herhaaldelik differensieer (met y-as $\uparrow$ en EI konst):

Note: If we differentiate repeatedly (with y-axis $\uparrow$ and EI const):

$$\begin{aligned} EI\frac{d^2y}{dx^2} &= M \\ EI\frac{d^3y}{dx^3} &= \frac{dM}{dx} = -S \\ EI\frac{d^4y}{dx^4} &= -\frac{dS}{dx} = -w(x) \end{aligned}$$

Met y-as / *With y-axis* $\downarrow \quad \Rightarrow \quad EI\frac{d^4y}{dx^4} = \frac{dS}{dx} = w(x)$

Tipiese randvoorwaardes / Typical boundary conditions:

Ingebed aan beide kante — *Embedded on both sides*

$x = 0$ $\qquad$ $x = L$

$$y(0) = 0,\ y'(0) = 0 \qquad y(L) = 0,\ y'(L) = 0$$

Ingebed links en vry regs — *Embedded left and free right*

Links: uitwyking & helling nul

Left: deflection & slope zero

$x = 0$ $\qquad$ $x = L$

Regs: M & S nul

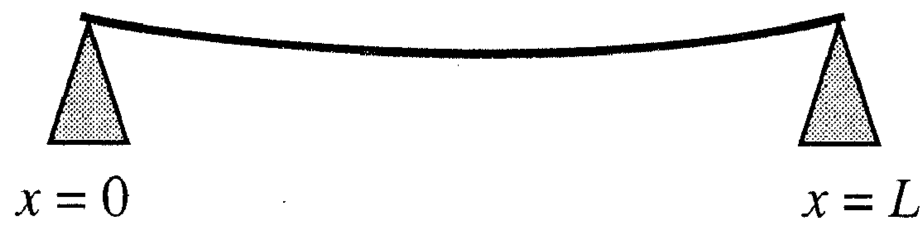
Right: M & S zero

$$y(0) = 0,\ y'(0) = 0 \qquad y''(L) = 0,\ y'''(L) = 0$$

Eenvoudig ondersteun aan beide kante *Simply supported on both sides*

Links: uitw & M nul
Left: defl & M zero

Regs: uitw & M nul
Right: defl & M zero



$$y(0) = 0,\ y''(0) = 0 \qquad y(L) = 0,\ y''(L) = 0$$

Voorbeeld 1 (bl 166): 'n Balk van lengte L is ingebed aan beide kante. Vind die uitwyking van die balk as 'n konstante las w_0 uniform versprei is oor sy lengte, m.a.w. $w(x) = w_0,\ 0 < x < L$. *Example 1 (page 166): A beam of length L is embedded on both ends. Find the deflection of the beam if a constant load w_0 is uniformly distributed along its length, i.e. $w(x) = w_0,\ 0 < x < L$.*

$$EI\frac{d^4y}{dx^4} = w_0$$

Antw/*Ans:* $y(x) = \frac{1}{24}\frac{w_0}{EI}x^2(x-L)^2$; **Max uitwyking/***deflection:* $y_{\max} = \frac{1}{384}\frac{w_0}{EI}L^4$

Voorbeeld 2 (Oef 3.9, Nr 3): Soortgelyk aan vb 1, maar ingebed links en eenvoudig ondersteun regs. *Example 2 (Ex 3.9, Nr 3): Similar to example 1, but embedded left and simply supported right.*

Antw/*Ans:* $y(x) = \dfrac{w_0}{48EI}\, x^2(x-L)(2x-3L)$

Voorbeeld 3: Soortgelyk aan vb 1, maar vry links en eenvoudig ondersteun regs. *Example 3: Similar to example 1, but free left and simply supported right.*

Antw/*Ans:* **Onoplosbaar**/*Unsolvable*