Example 1

$$y(0)=0\ ; \qquad y(L)=0$$
$$y'(0)=0\ ; \qquad y'(L)=0$$

$$EI\frac{d^4y}{dx^4} = w_0$$

$$\Rightarrow \frac{d^4y}{dx^4} = \frac{w_0}{EI} = A \quad \text{given}$$

$$\frac{d^3y}{dx^3} = Ax + B$$

$$\frac{d^2y}{dx^2} = \frac{1}{2}Ax^2 + Bx + C$$

$$y'(x) = \frac{dy}{dx} = \frac{1}{6}Ax^3 + \frac{1}{2}Bx^2 + Cx + \cancel{D} = 0$$

$$y'(0) = D = 0$$

$$y'(L) = \frac{1}{6}AL^3 + \frac{1}{2}BL^2 + CL = 0$$

$$\Rightarrow 2AL^2 + 6BL + 12C = 0 \quad ①$$

$$y(x) = \frac{1}{24}Ax^4 + \frac{1}{6}Bx^3 + \frac{1}{2}Cx^2 + \cancel{E} = 0$$

$$y(0) = E = 0$$

$$y(L) = \frac{1}{24}AL^4 + \frac{1}{6}BL^3 + \frac{1}{2}CL^2 = 0$$

$$\Rightarrow AL^2 + 4BL + 12C = 0 \quad ②$$

From ① and ②: $2AL^2 + 6BL = AL^2 + 4BL$

$$2B\cancel{L} = -AL^{\cancel{2}}$$

$$B = -\frac{AL}{2}$$

Into ②: $AL^2 + 4\left(-\frac{AL}{2}\right)L + 12C = 0$

$$AL^2 - 2AL^2 + 12C = 0$$

$$\Rightarrow C = \frac{AL^2}{12}$$

$$y(x) = \frac{1}{24}Ax^4 - \frac{1}{6}\left(\frac{AL}{2}\right)x^3 + \frac{1}{2}\left(\frac{AL^2}{12}\right)x^2$$

$$= \frac{1}{24}Ax^4 - \frac{1}{12}ALx^3 + \frac{1}{24}AL^2x^2$$

$$= \frac{Ax^2}{24}\left(x^2 - 2Lx + L^2\right)$$

$$= \frac{Ax^2}{24}(x-L)(x-L)$$

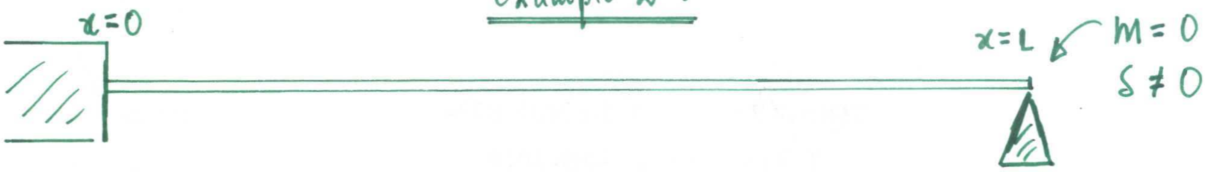
$$= \frac{1}{24}\frac{w_0}{EI}x^2(x-L)^2$$

$$y_{max} = y\left(\frac{L}{2}\right) = \frac{1}{24}\frac{w_0}{EI}\frac{L^2}{4}\left(\frac{L^2}{4}\right)$$

$$= \frac{1}{24}\frac{w_0}{EI}\frac{L^4}{16}$$

$$= \frac{1}{384}\frac{w_0}{EI}L^4$$

Example 2 :



$y(0)=0$ $\qquad$ $y(L)=0$

$y'(0)=0$ $\qquad$ $y''(L)=0$

$$EI\frac{d^4y}{dx^4} = \overset{=w_0}{w(x)}$$

$$\Rightarrow \frac{d^4y}{dx^4} = \overset{\text{given}}{A} \qquad A=\frac{w_0}{EI}$$

$$\frac{d^3y}{dx^3} = Ax+B$$

$$y''(x) = \frac{d^2y}{dx^2} = \frac12 Ax^2+Bx+C$$

$$\longrightarrow y''(L) = \boxed{\frac12 AL^2+BL+C=0} \quad ①$$

$$y'(x) = \frac16 Ax^3+\frac12 Bx^2+Cx+\overset{=0}{D}$$

$$\longrightarrow y'(0) = \boxed{D=0}$$

$$y(x) = \frac1{24}Ax^4+\frac16 Bx^3+\frac12 Cx^2+\overset{=0}{E}$$

$$\longrightarrow y(0) = \boxed{E=0}$$

$$\Rightarrow y(x) = \frac1{24}Ax^4+\frac16 Bx^3+\frac12 Cx^2$$

$$\longrightarrow y(L) = \frac1{24}AL^4+\frac16 BL^3+\frac12 CL^2=0$$

$$\Rightarrow \boxed{\frac1{24}AL^2+\frac16 BL+\frac12 C=0} \quad ②$$

$$①: \frac12 AL^2+BL+C=0$$

$$②\times 6: \frac14 AL^2+BL+3C=0$$

$$①-(②\times 6): \frac14 AL^2-2C=0 \Rightarrow C=\frac18 AL^2$$

$$BL = -\frac18 AL^2-\frac12 AL^2$$

$$= -\frac58 AL^2$$

$$B = -\frac58 AL$$

$$\Rightarrow y(x) = \frac1{24}Ax^4+\frac16\left(-\frac58 AL\right)x^3+\frac12\left(\frac18 AL^2\right)x^2$$

$$= \frac1{24}Ax^4-\frac5{48}ALx^3+\frac1{16}AL^2x^2$$

$$= \frac{Ax^2}{48}(2x^2-5Lx+3L^2)$$

$$= \frac{Ax^2}{48}(2x-3L)(x-L)$$

$$= \frac{w_0}{48EI}x^2(x-L)(2x-3L)$$

$y_{max} = ?$ (where $y'(x)=0$)

$$y'(x) = \frac16 Ax^3-\frac5{16}ALx^2+\frac18 AL^2x=0$$

when $x=0$

and $8Ax^2-15ALx+6AL^2=0$

$$\Rightarrow x = \frac{15AL\pm\sqrt{(15AL)^2-4(8A)(6AL^2)}}{2(8A)}$$

$$= \frac{15AL\pm AL\sqrt{225-192}}{16A}$$

$$= \frac{(15\pm\sqrt{33})L}{16}$$

$$= 1,297L;\ 0,578L$$

$$y_{max} = y(0,578L) = \dots$$

EXAMPLE 3

M=S=0

$y''(0) = 0$

$y'''(0) = 0$

$y(L) = 0$

$y''(L) = 0$

$$EI \frac{d^4y}{dx^4} = W(x) \quad w_0$$

given

$$\frac{d^4y}{dx^4} = A \quad ; \quad A = \frac{w_0}{EI}$$

$$y'''(x) = \frac{d^3y}{dx^3} = Ax + B \; {=0}$$

$$\longrightarrow y'''(0) = \boxed{B = 0}$$

$$\frac{d^3y}{dx^3} = Ax$$

$$y''(x) = \frac{d^2y}{dx^2} = \frac{1}{2}Ax^2 + C \; {=0}$$

$$\longrightarrow y''(0) = \boxed{C = 0}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{2}Ax^2$$

$$\rightarrow y''(L) = \frac{1}{2}AL^2 = 0$$

Not consistent with $A = \frac{w_0}{EI} \neq 0$

and $L \neq 0$

$\Rightarrow$ Unsolvable ?!