2.7: Toepassing 3: Saamgestelde rente

2.7: Application 3: Compound interest

2.7: Toepassing 3: Saamgestelde rente

Probleem: 'n Bedrag van P_0 word teen 'n rentekoers van x% per jaar belê.
Hoeveel geld is in die rekening na t jaar?

Rentekoers: $r = x\% = x/100$

Los die probleem op vir die scenarios waar die rentekoers (i) jaarliks, (ii) half-jaarliks, (iii) kwartaliks, (iv) daagliks en (v) kontinu saamgestel word

2.7: Application 3: Compound interest

Problem: An amount of P_0 is invested at an annual interest rate of x%.
How much money is in the account after t years?

Interest rate: $r = x\% = x/100$

Solve the problem for the scenarios where interest is compounded:

(i) annually, (ii) biannually, (iii) quarterly, (iv) daily, and (v) continuously

Met jaarlikse samestelling van rente:

With interest compounded annually:

Aanvanklik / *Initially:* $P(0) = P_0$

Na 1 **jaar** / *After* 1 *year:* $P(1) = P_0 + rP_0 = (1+r)P_0$

Na 2 **jaar** / *After* 2 *years:* $P(2) = (1+r)P(1) = (1+r)^2 P_0$

Na 3 **jaar** / *After* 3 *years:* $P(3) = (1+r)^3 P_0$

•

Na t **jaar** / *After* t *years:* $P(t) = (1+r)^t P_0$

Met jaarlikse samestelling van rente:

With interest compounded annually:

Aanvanklik / *Initially:* $P(0) = P_0$

Na 1 **jaar** / *After* 1 *year:* $P(1) = P_0 + rP_0 = (1+r)P_0$

Na 2 **jaar** / *After* 2 *years:* $P(2) = (1+r)P(1) = (1+r)^2 P_0$

Na 3 **jaar** / *After* 3 *years:* $P(3) = (1+r)^3 P_0$

•

Na t **jaar** / *After* t *years:* $P(t) = (1+r)^t P_0$

Met half-jaarlikse samestelling van rente:

With interest compounded biannually:

Aanvanklik / *Initially:* $P(0) = P_0$

Na $\frac{1}{2}$ **jaar** / *After* $\frac{1}{2}$ *year:* $P(\frac{1}{2}) = P_0 + \frac{r}{2}P_0 = \left(1 + \frac{r}{2}\right) P_0$

Na 1 **jaar** / *After* 1 *year:* $P(1) = (1+\frac{r}{2})P(\frac{1}{2}) = \left(1 + \frac{r}{2}\right)^2 P_0$

•

Na t **jaar** / *After* t *years:* $P(t) = \left(1 + \frac{r}{2}\right)^{2t} P_0$

Met kwartalikse samestelling van rente:

With interest compounded quarterly::

Na t **jaar /** *After* t *years:* $P(t) = \left(1 + \frac{r}{4}\right)^{4t} P_0$

Met kwartalikse samestelling van rente:

With interest compounded quarterly::

Na t **jaar /** *After* t *years:* $P(t) = \left(1 + \frac{r}{4}\right)^{4t} P_0$

Met daaglikse samestelling van rente:

With interest compounded daily:

Na t **jaar /** *After* t *years:* $P(t) = \left(1 + \frac{r}{365}\right)^{365t} P_0$

Met kwartalikse samestelling van rente:

With interest compounded quarterly::

Na t **jaar /** *After* t *years:* $P(t) = \left(1 + \frac{r}{4}\right)^{4t} P_0$

Met daaglikse samestelling van rente:

With interest compounded daily:

Na t **jaar /** *After* t *years:* $P(t) = \left(1 + \frac{r}{365}\right)^{365t} P_0$

Met kontinue samestelling van rente:

With interest compounded continuously:

Na t **jaar /** *After* t *years:* $P(t) = \left(1 + \frac{r}{n}\right)^{nt} P_0$, **met /** *with* $n \to \infty$

$$= \lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{nt} P_0$$

Stel / *Set:* $y = \left(1 + \frac{r}{n}\right)^{nt}$ **en /** *and* $s = r/n \Rightarrow n = r/s$

$$\Rightarrow \quad y = \left[(1+s)^{1/s}\right]^{rt}$$

As / *If* $n \to \infty$, **dan /** *then* $s \to 0$ **(** r **is konstant /** *constant* **)**

Laat / *Let* $f = (1+s)^{1/s}$ **sodat** / *so that* $\ln f = \dfrac{\ln(1+s)}{s}$

$$\Rightarrow \quad \lim_{s\to 0} \ln f = \lim_{s\to 0} \frac{\ln(1+s)}{s}$$

$$= \lim_{s\to 0} \frac{1}{1+s} \quad \textbf{(L'Hopital)}$$

$$= 1$$

$$\Rightarrow \quad e^{\lim_{s\to 0} \ln f} = \lim_{s\to 0} e^{\ln f} = \lim_{s\to 0} f = e^1 = e$$

$$\Rightarrow \quad \lim_{s\to 0} (1+s)^{1/s} = e$$

$$\Rightarrow \quad \lim_{n\to\infty} \left(1 + \frac{r}{n}\right)^{nt} = \left[\lim_{s\to 0} (1+s)^{1/s}\right]^{rt}$$

$$= e^{rt}$$

Dus vir kontinue samestelling van rente:

Therefore, for interest compounded continuously:

$$\boxed{P(t) = P_0 e^{rt}}$$

Dus vir kontinue samestelling van rente:

Therefore, for interest compounded continuously:

$$\boxed{P(t) = P_0 e^{rt}}$$

Dit bevredig die DV:

This satisfies the DE:

$$\boxed{\frac{dP}{dt} = rP \quad \textbf{met} \ /\textit{with} \quad P(0) = P_0}$$

Voorbeeld: **'n Bedrag van R 1000 word vir 3 jaar teen 'n rentekoers van 6% per jaar belê. Hoeveel is dit werd: (a) as rente jaarliks bygevoeg word en (b) as rente kontinu saamgestel word?**

(SELFSTUDIE) Toepassing 4: Newton se wet van afkoeling (pp 20 & 75)

Example: **An amount of R 1000 is invested for 3 years at an annual interest rate of 6%. How much is it worth: (a) when the interest is compounded annually, and (b) when the interest is compounded continuously?**

(SELF STUDY) Application 4: Newton's law of cooling (pp 20 & 75)

Voorbeeld: 'n Bedrag van R 1000 word vir 3 jaar teen 'n rentekoers van 6% per jaar belê. Hoeveel is dit werd: (a) as rente jaarliks bygevoeg word en (b) as rente kontinu saamgestel word?

Antwoorde: (a) R 1191 (b) R 1197

(SELFSTUDIE) Toepassing 4: Newton se wet van afkoeling (pp 20 & 75)

Example: An amount of R 1000 is invested for 3 years at an annual interest rate of 6%. How much is it worth: (a) when the interest is compounded annually, and (b) when the interest is compounded continuously?

Answers: (a) R 1191 (b) R 1197

(SELF STUDY) Application 4: Newton's law of cooling (pp 20 & 75)