Kettinglyne II

Catenaries II

Probleem/*Problem* **1: Gegee/***Given* ℓ **&** h**, vind/***find* a **&** c

$$\boxed{h} = c[\cosh(a/c) - 1] \tag{1}$$

$$\boxed{\ell} = c\sinh(a/c) \tag{2}$$

Vanuit/*From* (1)**:**

$$h + c = c\cosh(a/c) \tag{3}$$

$(3)^2 - (2)^2$**:**

$$c = \frac{\ell^2 - h^2}{2h}$$

Stel terug in / *Substitute back into* (2)**:**

$$a = c\,\mathrm{arcsinh}(\ell/c)$$

Voorbeeld: Bereken a **en** c **wanneer die lengte van die gedeflekteerde kabel gelyk is aan** 3.3 **m en die insakking gelyk is aan** 0.6 **m.** *Example: Calculate* a *and* c *when the length of the deflected cable equals* 3.3 *m and the sag equals* 0.6 *m.*

Antwoord/*Answer:* $a = 1.5$ **m;** $c = 1.9688$ **m**

Probleem/*Problem* **2: Gegee/***Given* a **&** h**, vind/***find* ℓ **&** c

$$\boxed{h} = c[\cosh(\boxed{a}/c) - 1] \tag{4}$$

$$\ell = c\sinh(\boxed{a}/c) \tag{5}$$

Vergelyking (4)**: 1 vgl, 1 onbekende /** *Equation* (4)*: 1 eqn, 1 unknown*

Opl-opsies/*Sol options:* **(1)** Numeries (meer akkuraat)/*Numerical (more accurate)*
(2) Formule vir klein insakking/*Formula for small sag*
(benadering/*approximation:* $\cosh x \approx 1 + \frac{x^2}{2!}$**)**

Voorbeeld: Die span en insakking van 'n elastiese kabel is 150 m en 60 m, onderskeidelik. Die kabel het 'n massa per eenheidslengte van 12 kg/m. Bepaal
(i) die trekkrag T_1 by die onderpunt van die gedeflekteerde kabel,
(ii) die maksimum trekkrag in die gedeflekteerde kabel, en
(iii) die lengte van die gedeflekteerde kabel.

Example: *The span and sag of a flexible cable are* 150 *m and* 60 *m, respectively. The cable has a mass per unit length of* 12 *kg/m. Determine*
(i) the tension T_1 *at the lowest point of the deflected cable,*
(ii) the maximum tension in the deflected cable, and
(iii) the length of the deflected cable.

$$60 = c\,(\cosh(150/c) - 1)$$

Oplossings-opsie 2: Aanvaar dat die insakking klein is / *Solution option 2: Assume that the sag is small* **(**$c = \dfrac{T_1}{\rho}$ **groot/***large***)**

$$\Rightarrow \quad 60 = c\left(1 + \frac{1}{2}(150/c)^2 - 1\right)$$

$$\Rightarrow \quad c = 187.5\,\textbf{m} \quad \textbf{(korrekte waarde/}\textit{correct value}\ 196.8\,\textbf{m)}$$

$$T_1 = c\rho = 187.5 \times 12 \times 9.81 = 22\,073\,\textbf{N}$$

(korrekte waarde/*correct value* $23\,167$ **N)**

$$T_2 = T_1 + \rho h = 22\,073 + (12)(9.81)(60) = 29\,136\,\textbf{N}$$

(korrekte waarde/*correct value* $30\,230$ **N)**

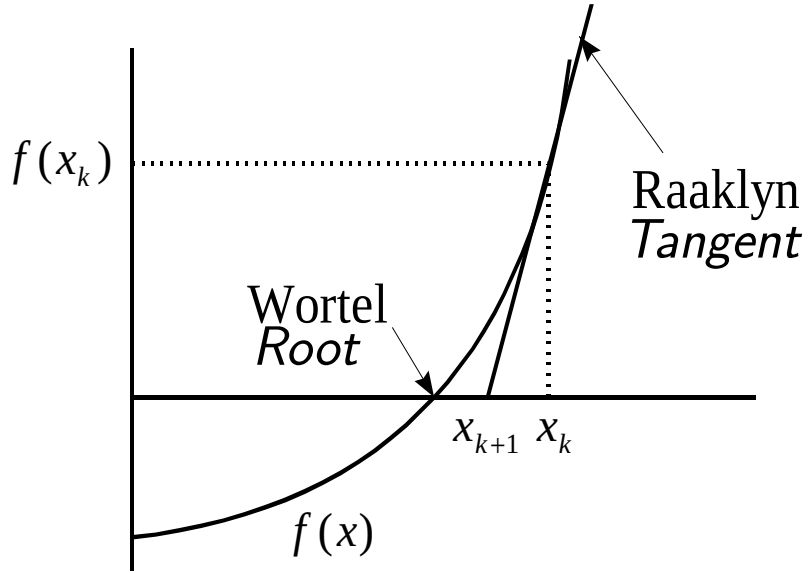
$$\ell = 187.5\,\sinh(150/187.5) = 166.5\,\textbf{m}$$

Die lengte is dus/*The length is therefore* $333\,\textbf{m}$ **(korrekte waarde/***correct value* $330\,\textbf{m}$**)**

Oplossings-opsie 1: Numeries (meer akkuraat)

Solution option 1: Numerical (more accurate)

Newton se metode / *Newton's method* $f(x) = 0$



$$f'(x_k) = \frac{f(x_k) - 0}{x_k - x_{k+1}}$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_0, \quad x_1, \quad x_2, \quad x_3 \quad \rightarrow \quad \textbf{wortel/}\textit{root}$$

$$60 = c\,(\cosh(150/c) - 1)$$

$$f(c) = c\,(\cosh(150/c) - 1) - 60 = 0$$

$$f'(c) = (\cosh(150/c) - 1) - \frac{150}{c}(\sinh(150/c))$$

Kies/*Choose* $c_0 = 187.5$ **(benadering vir klein insakking/***approximation for small sag***)**

$$c_1 = c_0 - \frac{f(c_0)}{f'(c_0)} = 196.3$$

$$c_2 = 196.8; \quad c_3 = 196.8$$

$$\Rightarrow c = 196.8\,\mathsf{m}$$

Matlab-afvoer/*output:*

Iterasie/*Iteration*	c	$f(c)$	$f'(c)$
0	187.5000	3.2691	-0.3730
1	196.2630	0.1661	-0.3361
2	196.7574	0.0005	-0.3342
3	196.7588	0.0000	-0.3342

Probleem/*Problem* 3: Gegee/*Given* a & ℓ, vind/*find* h & c

$$h = c[\cosh(\boxed{a}/c) - 1] \tag{6}$$

$$\boxed{\ell} = c\sinh(\boxed{a}/c) \tag{7}$$

Voorbeeld/Example: Gegee/*Given* $a = 5$m en/*and* $\ell = 20$m. Vind/*Find* h en/*and* c.

$$20 = c\,\sinh(5/c)$$

Benadering vir klein insakking / *Approximation for small sag:* $\sinh(x) \approx x + \dfrac{x^3}{3!}$

$$20 = c\left(\frac{5}{c} + \frac{1}{6}\left(\frac{5}{c}\right)^3\right)$$

$$\Rightarrow c = 1.1785\,\text{m}$$

$$h = 1.1785(\cosh(5/1.1785) - 1) \approx 39\,\text{m}$$

Probeer eerder Newton se metode... / *Rather try Newton's method...*

$$20 = c\,\sinh(5/c)$$

$$f(c) = c\,\sinh(5/c) - 20 = 0$$

$$f'(c) = \sinh(5/c) - (5/c)\cosh(5/c)$$

Kies/*Choose* $c_0 = 1.1785$ **(benadering vir klein insakking/***approximation for small sag***)**

$$c_1 = c_0 - \frac{f(c_0)}{f'(c_0)} = 1.3645$$

Herhaal... / *Repeat...* $\Rightarrow\ c = 1.5320\,\mathsf{m}$

$$h = 1.5320\,(\cosh(5/1.5320) - 1) = 18.53\,\mathsf{m}$$

Matlab-afvoer/*output:*

Iterasie/*Iteration*	c	$f(c)$	$f'(c)$
0	1.1785	20.999	-112.873
1	1.3645	6.6086	-52.0460
⋮	⋮	⋮	⋮
5	1.5320	0.0000	-29.6791