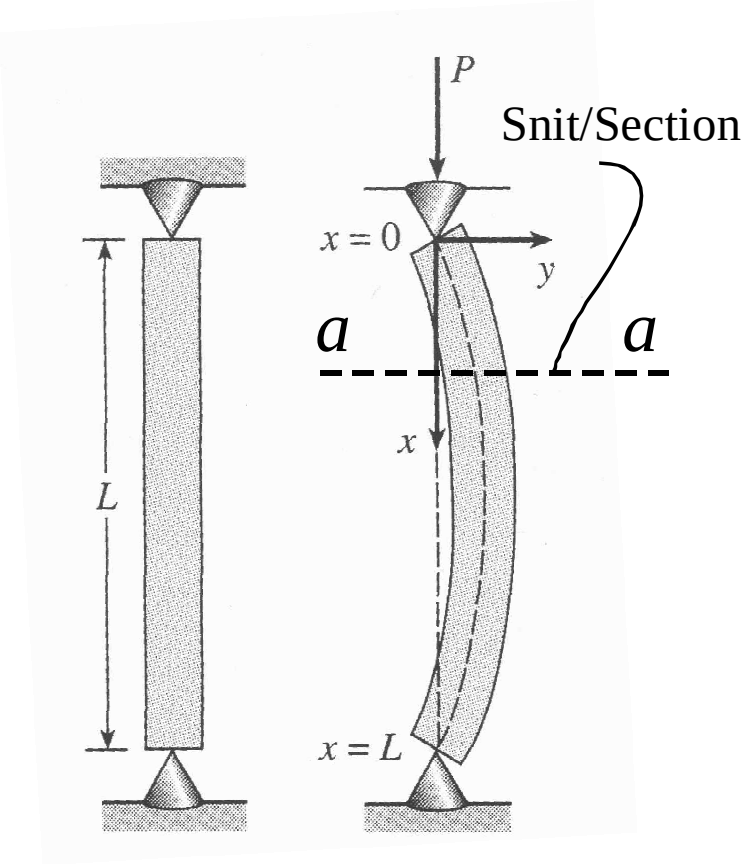
3.9 Knikking van 'n kolom

(bladsy 169)

Probleem: Vir watter waarde(s) van P sal die kolom (pilaar) knik?



3.9 Buckling of a column

(page 169)

Problem: For which value(s) of P will the column (pillar) buckle?

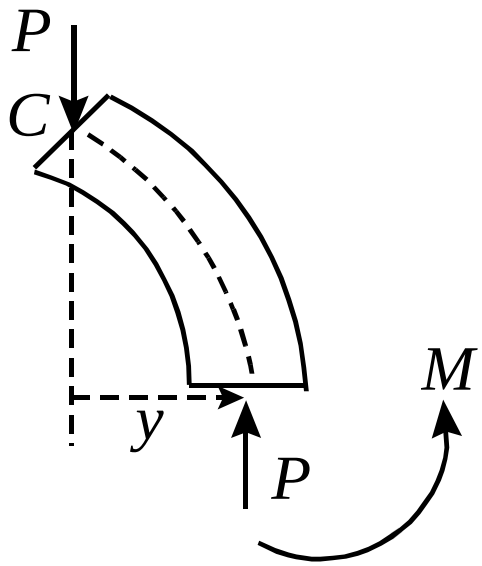
Aannames:

(1) Die kolom is perfek reguit voor die lading P aangwend word
(2) Die materiaal is homogeen
(3) Die uitwykings is klein

Assumptions:

(1) The column is perfectly straight before the load P is applied
(2) The material is homogeneous
(3) The deflection is small

Beskou VDD van kolom bo snit $a-a$ / *Consider FBD of column above section* $a-a$*:*



$$\boxed{\Sigma M_C = 0} \quad \Rightarrow \quad M + Py = 0$$

Maar/*But* $M = EI\dfrac{d^2y}{dx^2}$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \lambda y = 0 \quad \textbf{met/}\textit{with} \quad \lambda = \frac{P}{EI}$$

Randvoorwaardes/*Boundary conditions* $y(0) = 0$ **en/***and* $y(L) = 0$

Los op soos in geval III in vorige lesing / *Solve as in case III of previous lecture:*

$$\lambda_n = \frac{n^2\pi^2}{L^2} = \frac{P_n}{EI}$$

$$P_n = \frac{n^2\pi^2}{L^2}EI,\ n = 1, 2, \ldots \textbf{ (Kritieke ladings/}\textit{Critical loads}\textbf{)}$$

$$y_n(x) = C\sin\left(\frac{n\pi}{L}x\right)$$

Die konstante C is onbepaald, maar klein / *The constant C is undetermined, but small*

Vir/*For* $n = 1$ **Eerste kritieke lading/***First critical load* — **Euler lading/***load*

$$P_1 = \frac{\pi^2}{L^2}EI$$

Kleinste lading benodig vir knikking met geen ondersteuning /
Smallest load required for buckling with no physical restraints

$$y_1(x) = C\sin\left(\frac{\pi x}{L}\right) \textbf{ (1e knikkingsmodus/}\textit{1st buckling mode}\textbf{)}$$

Vir/*For* $n = 2$ **Tweede kritieke lading/***Second critical load*

$$P_2 = \frac{4\pi^2}{L^2} EI$$

Kleinste lading benodig vir knikking met ondersteuning by $x = L/2$
Smallest load required for buckling with physical restraint at $x = L/2$

$$y_2(x) = C \sin\left(\frac{2\pi x}{L}\right)$$ **(2e knikkingsmodus/***2nd buckling mode***)**

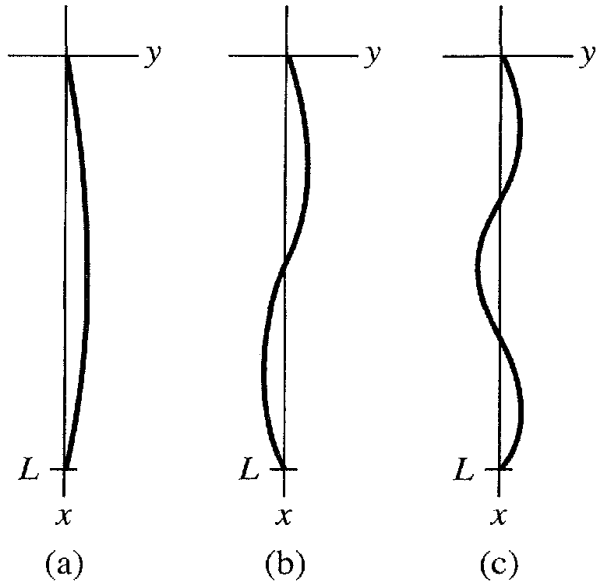


Figure 3.44 Deflection curves for compressive forces P_1, P_2, P_3