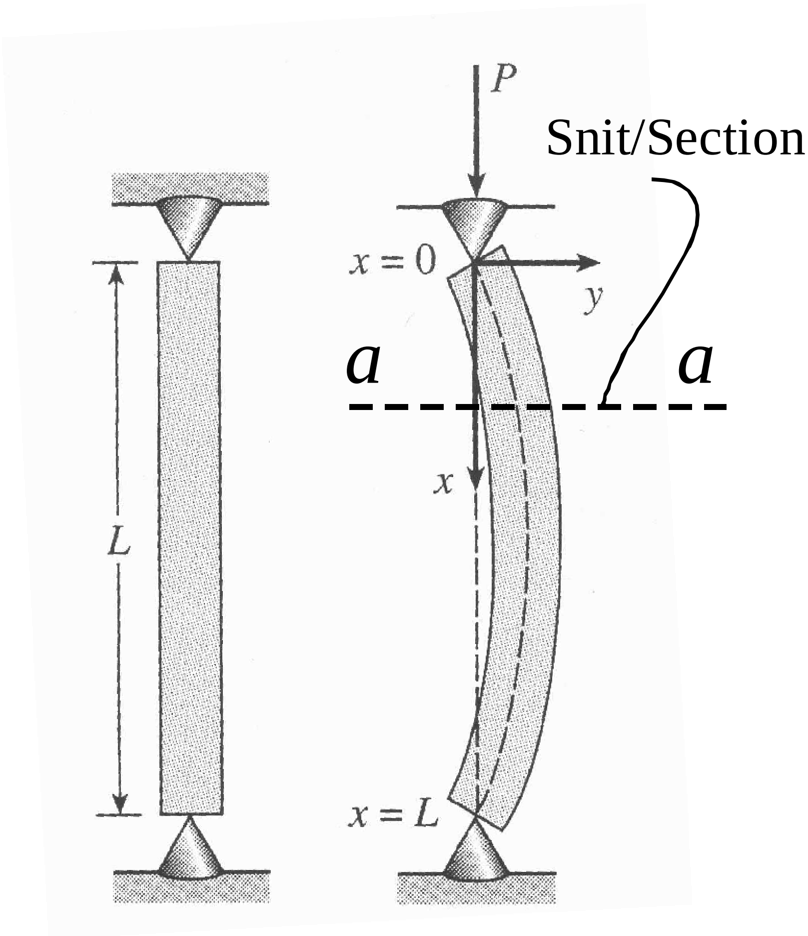
3.9 Buckling of a column *(p 169)*

Problem: For which value(s) of P will the column (pillar) buckle?

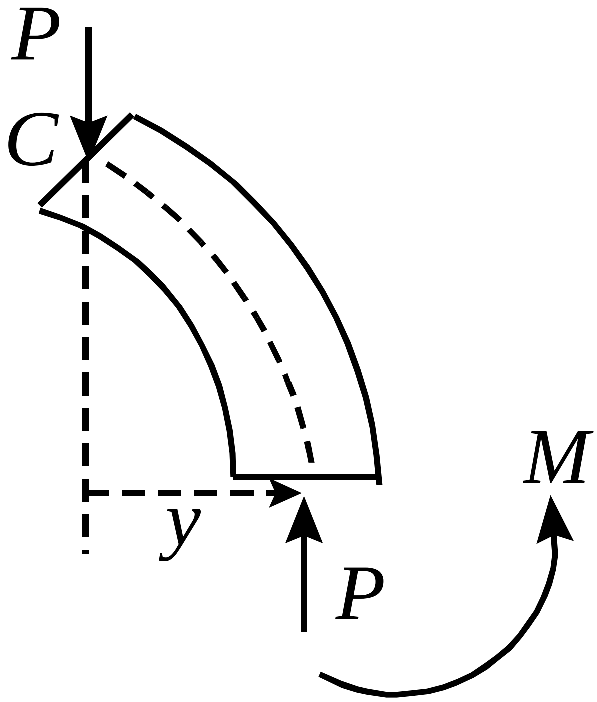


Assumptions:

(1) The column is perfectly straight before the load P is applied
(2) The material is homogeneous
(3) The deflection is small

Buckling of a column Tr 2

Consider FBD of column above section $a - a$:



$$\boxed{\Sigma M_C = 0} \quad \Rightarrow \quad M + Py = 0$$

$$\text{But} \quad M = EI\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \lambda y = 0 \quad \text{with} \quad \lambda = \frac{P}{EI}$$

Boundary conditions: $y(0) = 0$ and $y(L) = 0$

Buckling of a column

Solve as in case III of previous lecture:

$$\lambda_n = \frac{n^2\pi^2}{L^2} = \frac{P_n}{EI}$$

$$P_n = \frac{n^2\pi^2}{L^2}EI,\ n = 1, 2, \ldots \quad \textbf{(Critical loads)}$$

$$y_n(x) = C\sin\left(\frac{n\pi}{L}x\right)$$

The constant C is undetermined, but small

$\boxed{\text{For } n = 1}$ First critical load — **Euler load**:

$$P_1 = \frac{\pi^2}{L^2}EI$$

Smallest load required for buckling with no physical restraints

$$y_1(x) = C\sin\left(\frac{\pi x}{L}\right) \quad \textbf{(1st buckling mode)}$$

For $n = 2$ Second buckling mode:

$$P_2 = \frac{4\pi^2}{L^2} EI$$

Smallest load required for buckling with physical restraint at $x = L/2$

$$y_2(x) = C \sin\left(\frac{2\pi x}{L}\right) \textbf{ (2nd buckling mode)}$$

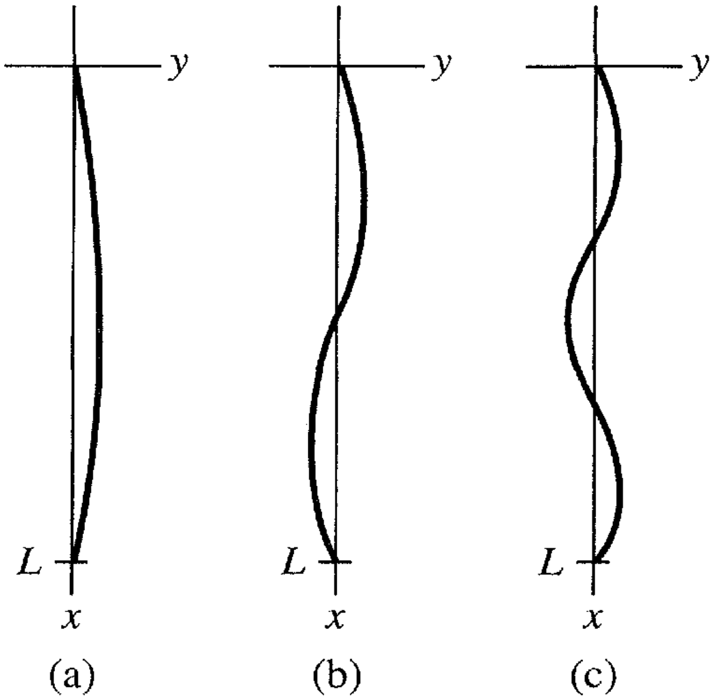


Figure 3.44 Deflection curves for compressive forces P_1, P_2, P_3