STELLENBOSCH UNIVERSITY

APPLIED MATHEMATICS AM B242

VECTOR ANALYSIS

OUTCOMES

The module outcomes are labeled here according to the corresponding sections in the handbook ZILL & WRIGHT. On successful completion of the module, the student will be acquainted with the material in the following sections in ZILL & WRIGHT: 7.5, 9.1, 9.4, 9.5, 9.6, 9.7, 9.8, 9.9, 9.10, 9.11, 9.12, 9.13, 9.14, 9.15, 9.16.

In particular, he/she will be able to perform the following:

7.5

- Be able to write down the equation of a line in 3D in vector form, or in parametric form, or in symmetric form when information such as (a) one point on the line and information about the direction, or (b) two points on the line, are given.
- Be able to write down the equation of a plane in 3D in vector form with two orientation vectors (i.e. $\mathbf{r} = \mathbf{r}_1 + t\mathbf{a} + s\mathbf{b}$), or with a normal vector $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_1$, or in the form of an equation in x, y, and z, when information such as (a) one point on the plane and two orientation vectors are given, or (b) one point on the plane and the normal to the plane are given, or (c) three points on the plane are given, or (d) one point on the plane and other information from which the orientation of the plane may be determined, is given.

9.1

- Be able to draw a vector function $\mathbf{r}(t)$ in 3D (simple examples that can reasonably be drawn).
- Be able to calculate the derivative of a vector function $\mathbf{r}'(t)$ en be able to use it for example to find the tangent line at certain points on $\mathbf{r}(t)$.
- Be able to apply the chain rule for partial derivatives correctly.
- Be able to calculate the arc length of $\mathbf{r}(t)$ on a given interval of t.

9.4

- Be able to sketch level curves (contours) of $f(x, y)$.
- Be able to describe level surfaces of $f(x, y, z)$ (only simpler cases where the level surface is a sphere, ellipsoid, or paraboloid).

9.5

- Be able to calculate the gradient of $f(x, y)$ and of $f(x, y, z)$.
- Be able to calculate the directional derivative of $f(x, y)$ in a given direction.
- Be able to prove that the maximum of the directional derivative is given by $\|\nabla f\|$.
- Must be able to prove identities such as Exercise 9.5, 45 to 48.

9.6

- Be able to give the interpretation of ∇f and to use it in problems: it is a vector that points in the direction of greatest increase in f, and of which the magnitude is the rate of increase of f in that direction.

- Be able to use ∇f to (a) find the equation of the line perpendicular to a level curve, (b) find the equation of the line perpendicular to a level surface, and (c) to find the equation of tangent plane to a level surface.

9.7
- Be able sketch vector fields in 2D. Only fairly simple 'drawable' cases may be required.
- Be able to calculate $\nabla \cdot \mathbf{F}$ as well as $\nabla \times \mathbf{F}$ for $\mathbf{F}(x, y, z) = \mathbf{i}P(x, y, z) + \mathbf{j}Q(x, y, z) + \mathbf{k}R(x, y, z)$, (a) in general, i.e. in terms of x, y, and z, as well as (b) at a specific (given) point.
- Be able to describe the terms *irrotational* and *incompressible.*
- Be able to prove identities such as Exercise 9.7, 17-32.

9.8
- Be able to calculate line integrals such as $\int F ds$, $\int \mathbf{F} \cdot d\mathbf{r}$ along given piecewise continuous paths. 2D and 3D cases may both be asked.
- Be able to calculate the work done on a particle as it is moved on a given path under the influence of a given force field.

9.9
- Be able to determine whether a differential of the form $Pdx + Qdy$ is an exact differential.
- Be able to obtain the potential function of an exact differential of the form $Pdx + Qdy$ by means of integration.
- Be able to calculate line integrals of the form $\int_C Pdx + Qdy$ in a path independent way by means of the potential function, where $Pdx + Qdy$ is an exact differential.
- Be able to show that $\mathbf{F} = \nabla f$ (i.e. $\mathbf{F}$ is a gradient field) can always be integrated in a path independent way, i.e. $\mathbf{F} \cdot d\mathbf{r}$ is an exact differential.
- Be able to determine whether the differential of the form $Pdx + Qdy + Rdz$ is an exact differential.
- Be able to find the potential function of the exact differential of the form $Pdx + Qdy + Rdz$ by means of integration.
- Be able to calculate line integrals of the form $\int_C Pdx + Qdy + Rdz$ in a path independent way by using the relevant potential function, where $Pdx + Qdy + Rdz$ is an exact differential.

9.10
- Be able to calculate double integrals (both type I and II) where either (a) the boundary is given by means of a figure, or (b) the boundary is given by means of equations.
- Be able to exchange the order of integration correctly where necessary or when asked.
- Be able to calculate the area and volume of bodies using $A = \iint_R dA$, or $V = \iint_R f(x, y) dA$.
- Be able to calculate masses, and coordinates of mass centers of laminas with variable density using: $m = \iint_R \rho(x, y) dA$, $m\bar{x} = \iint_R x\rho(x, y) dA$, $m\bar{y} = \iint_R y\rho(x, y) dA$.

— *Omit: Moments of inertia.*

9.11
- Be able to convert Cartesian coordinates to polar coordinates and to convert polar coordinates back to Cartesian coordinates.

- Know the area element in polar coordinates ($dA = rdrd\theta$) off by heart.
- Be able to calculate double integrals in polar coordinates and must be able to calculate areas, volumes, masses, mass centers and moments of inertia in polar coordinates.

9.12
- Know and be able to prove Green's theorem for a simple region that may be considered to be simultaneously of type I and type II.
- Be able to calculate both sides of Green's theorem (and hence verify the theorem). Line integrals must be parameterized correctly. If required, surface integrals must also be calculated in polar coordinates.

— *Omit: Regions containing holes as well as regions where the function is not sufficiently differentiable (e.g. Examples 4, 5, and 6.)*

9.13
- Know the expression for the surface element ($dS = \left(\sqrt{1 + (f_x)^2 + (f_y)^2}\right) dA$). (*Not necessary to be able to derive it.*).
- Be able to calculate the surface area of a given surface.
- Be able to calculate surface integrals of the form $\iint_S G(x, y, z)dS$.
- Be able to calculate the mass of a curved surface that has variable density.
- Be able to calculate the flux of a given vector field $\mathbf{F}$ through a given surface S ($\iint_S \mathbf{F} \cdot \mathbf{n}dS$.)

9.14
- Know Stokes' theorem. (*Not necessary to be able to prove it.*).
- Be able to calculate both sides of Stokes' theorem (and hence verify the theorem). Line integrals must be parameterized correctly. For surface integrals only projection onto the xy-plane will be required. (*Therefore, leave out projection onto the xz-, or yz-planes.*)

9.15
- Be able to calculate triple integrals where either (a) the boundary is given in a figure, or (b) the boundary is given by means of equations.
- Be able to interchange the order of integration correctly where necessary or when asked.
- Be able to calculate the volume of bodies from $V = \iiint_D dV$.
- Be able to calculate masses and coordinates of mass centers of bodies with variable density using: $m = \iiint_D \rho(x, y, z)dV$, $m\bar{x} = \iiint_D x\rho(x, y, z)dV$, $m\bar{y} = \iiint_D y\rho(x, y, z)dV$, $m\bar{z} = \iiint_D z\rho(x, y, z)dV$.

— *Omit: Moments of inertia that require triple integration.*

- Be able to convert cylindrical coordinates to Cartesian and back.
- Know the expression of the volume element ($dV = rdrd\theta dz$) of cylindrical coordinate and be able to use it to calculate triple integrals in cylindrical coordinates.
- Be able to convert spherical coordinates to Cartesian and back.
- Know the expression of the volume element ($dV = r^2 \sin\phi drd\phi d\theta$) of spherical coordinates and be able to use it to calculate triple integrals in spherical coordinates.

9.16
- Know the divergence theorem (Gauss' theorem). (*Not necessary to be able to prove it.*)

- Be able to calculate both sides of the divergence theorem (and hence verify the theorem), or be able to calculate one side, given the other.

General

Know the following terms (and be able to express in symbols where appropriate, e.g. '$\mathbf{F}$ is incompressible' means $\nabla \cdot \mathbf{F} = 0$): 'vector field', 'tangent line', 'tangent plane', 'gradient', 'rotation', 'curl', 'divergence', 'irrotational', 'incompressible', 'exact differential', 'path independent', 'normal on a surface', 'arc length', 'surface element', 'circulation around a closed curve', 'volume element', 'positively orientated closed path', 'flux through a surface', 'flux out of a volume', 'conservative field', 'work done'.