Applied Maths B242 TEST 1 - PREPARATION 2017

The test covers the following sections in ZILL & WRIGHT: 7.5, 9.1, 9.4, 9.5, 9.6, 9.7, 9.8, 9.9. No formula page will be given — formulae must be known by heart.

The test consists of two sections — Section A: multiple choice questions, and Section B: problems that must be written out.

For this test you must be able to do the following:

7.5

- Must be able to write down the equation of a line in 3D in vector form, or in parametric form, or in symmetric form when information such as (a) one point on the line and information about the direction, or (b) two points on the line, are given.
- Must be able to write down the equation of a plane in 3D in vector form with two orientation vectors (i.e. $\mathbf{r} = \mathbf{r}_1 + t\mathbf{a} + s\mathbf{b}$), or with a normal vector $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_1$, or in the form of an equation in x, y, and z, when information such as (a) one point on the plane and two orientation vectors are given, or (b) one point on the plane and the normal to the plane are given, or (c) three points on the plane are given, or (d) one point on the plane and other information from which the orientation of the plane may be determined, is given.

9.1

- Must be able to draw a vector function $\mathbf{r}(t)$ in 3D (simple examples that can reasonably be drawn).
- Must be able to calculate the derivative of a vector function $\mathbf{r}'(t)$ en be able to use it for example to find the tangent line at certain points on $\mathbf{r}(t)$.
- Must be able to apply the chain rule for partial derivatives correctly.
- Must be able to calculate the arc length of $\mathbf{r}(t)$ on a given interval of t.

9.4

- Must be able to sketch level curves (contours) of $f(x, y)$.
- Must be able to describe level surfaces of $f(x, y, z)$ (only simpler cases where the level surface is a sphere, an ellipsoid, a cone, a hyperboloid or a paraboloid).

9.5

- Must be able to calculate the gradient of $f(x, y)$ and of $f(x, y, z)$.
- Must be able to calculate the directional derivative of $f(x, y)$ and also of $f(x, y, z)$ in a given direction.
- Must be able to prove that the maximum of the directional derivative is given by $\|\nabla f\|$.
- Must be able to prove the following identities (and possibly similar ones): $\nabla(cf) = c\nabla f$, $\nabla(f + g) = \nabla f + \nabla g$, $\nabla(fg) = f\nabla g + g\nabla f$, en $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$. Here f and g are functions of x, y and z, and c is a constant.

9.6

- Must be able to give the interpretation of ∇f and to use it in problems: it is a vector that points in the direction of greatest increase in f, and of which the magnitude is the rate of increase of f in that direction.
- Must be able to use ∇f to (a) find the equation of the line perpendicular to a level curve, (b) find the equation of the line perpendicular to a level surface, and (c) to find the equation of tangent plane to a level surface.

9.7

- Must be able sketch vector fields in 2D. Only fairly simple 'drawable' cases may be required.
- Must be able to calculate $\nabla \cdot \mathbf{F}$ as well as $\nabla \times \mathbf{F}$ for $\mathbf{F}(x, y, z) = \mathbf{i}P(x, y, z) + \mathbf{j}Q(x, y, z) + \mathbf{k}R(x, y, z)$, (a) in general, i.e. in terms of x, y, and z, as well as (b) at a specific (given) point.
- Must be able to describe the terms *irrotational* and *incompressible*.
- Must be able to prove identities such as Exercise 9.7, 17-32. You must be able to prove similar other identities.

9.8

- Must be able to calculate line integrals such as $\int F ds$, $\int \mathbf{F} \cdot d\mathbf{r}$ along given piecewise continuous paths. 2D and 3D cases may both be asked.
- Must be able to calculate the work done on a particle as it is moved on a given path under the influence of a given force field.

9.9

- Must be able to determine whether a differential of the form $Pdx + Qdy$ is an exact differential.
- Must be able to obtain the potential function of an exact differential of the form $Pdx + Qdy$ by means of integration.
- Must be able to calculate line integrals of the form $\int_C Pdx + Qdy$ in a path independent way by means of the potential function, where $Pdx + Qdy$ is an exact differential.
