# TWB 242 Asses. 1 - Memo 2017

## Section A: out of 48.

, : 48
=b2,d4,b4,e2,d4,e4,c4,d4,e2,b2,a2,e2,e2,e4,a2,b4,

VEELVULDIGE KEUSE ANTWOORDBLAD — *MULTIPLE CHOICE ANSWER SHEET*

Naam / *Name*: MEMO Vc 2017 B242

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1. | a | **b** | c | d | e | 11. | **a** | b | c | d | e |
| 2. | a | b | c | **d** | e | 12. | a | b | c | d | **e** |
| 3. | a | **b** | c | d | e | 13. | a | b | c | d | **e** |
| 4. | a | b | c | d | **e** | 14. | a | b | c | d | **e** |
| 5. | a | b | c | **d** | e | 15. | **a** | b | c | d | e |
| 6. | a | b | c | d | **e** | 16. | a | **b** | c | d | e |
| 7. | a | b | **c** | d | e | 17. | a | b | c | d | e |
| 8. | a | b | c | **d** | e | 18. | a | b | c | d | e |
| 9. | a | b | c | d | **e** | 19. | a | b | c | d | e |
| 10. | a | **b** | c | d | e | 20. | a | b | c | d | e |

Studentenommer *Student number*

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **0** | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | **1** | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | **2** | 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | **3** | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | **4** | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 | **5** | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | **6** | 6 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | **7** |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |

Skryf u studentenommer in die spasies regs bo, en kleur daarna elke blokkie wat met die syfer ooreenstem swart in.

Vir u antwoorde, merk u keuse deur die blokkie in te kleur. Gebruik potlood.

*Write your student number in the spaces provided and then colour in the block corresponding to the digit.*

*For your answers, mark your choice by colouring in the block. Use pencil.*

## Section B: out of 38

## Total: 86 marks.

| **Afdeling B (38 punte)** | **Section B (38 marks)** |
|---|---|
| Beantwoord vrae B1 tot B4 op die vraestel. | *Answer questions B1 to B4 on the question paper.* |

B1 Laat $\mathbf{F}(x,y,z)$ 'n vektorveld wees, $g(x,y,z)$ 'n skalaarfunksie en $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$. Brei die volgende uit en vereenvoudig.

*Let $\mathbf{F}(x,y,z)$ be a vector field, let $g(x,y,z)$ be a scalar function, and $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$. Expand the following and simplify.*

10/10

**(a)** $\nabla \cdot (\nabla \times \mathbf{F})$

**(b)** $\nabla \cdot (g\mathbf{r})$

Wenk: Skryf $\mathbf{F}$ as $P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$.

*Hint: Write $\mathbf{F}$ as $P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$.* [10]

(a) $\underline{\nabla} \cdot (\underline{\nabla} \times \underline{F}) = \underline{\nabla} \cdot \left( \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \times \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \right) = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \cdot \begin{bmatrix} R_y - Q_z \\ P_z - R_x \\ Q_x - P_y \end{bmatrix}$

$= \frac{\partial}{\partial x}(R_y - Q_z) + \frac{\partial}{\partial y}(P_z - R_x) + \frac{\partial}{\partial z}(Q_x - P_y)$

$= R_{yx} - Q_{zx} + P_{zy} - R_{xy} + Q_{xz} - P_{yz}$

and if $\underline{F}$ is smooth to second order

$= R_{yx} - Q_{zx} + P_{zy} - R_{yx} + Q_{zx} - P_{zy}$

$= 0$ 5

(b) $\underline{\nabla} \cdot (g\underline{r}) = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} \cdot \begin{bmatrix} gx \\ gy \\ gz \end{bmatrix} = \frac{\partial}{\partial x}(gx) + \frac{\partial}{\partial y}(gy) + \frac{\partial}{\partial z}(gz)$

$= g_x x + g \cdot 1 + g_y y + g \cdot 1 + g_z z + g \cdot 1$

$= 3g + \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 3g + \underline{\nabla} g \cdot \underline{r}$ 5

B2 (a) Maak 'n skets van die vektorfunksie $\mathbf{r}(t)$, slegs vir $t \in [0,1]$. (b) Vind dan die simmetriese vorm van die vergelykings van die raaklyn aan $\mathbf{r}(t)$ by $t = \frac{1}{2}$. (c) Waar sny hierdie raaklyn die $z$-as ?

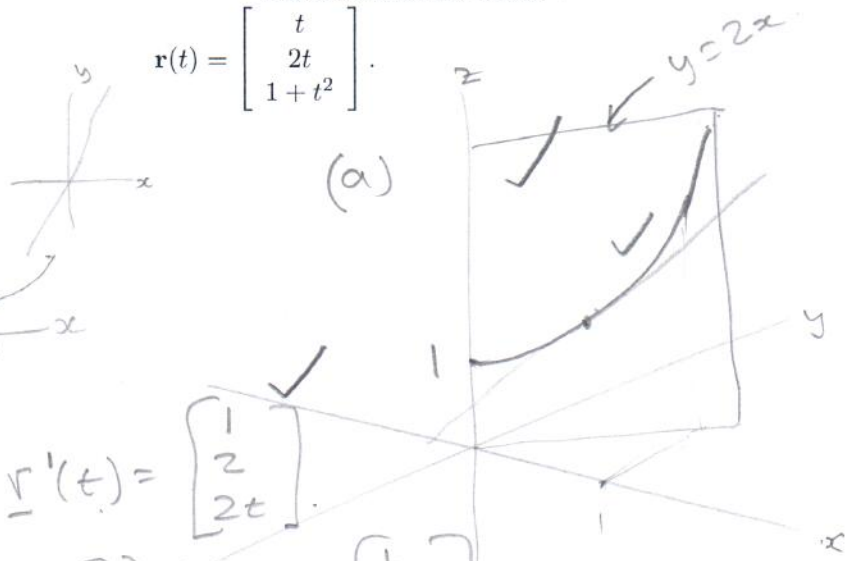
*(a) Draw a sketch of the vector function $\mathbf{r}(t)$, only for $t \in [0,1]$. (b) Then find the symmetric form of the equations of the tangent line to $\mathbf{r}(t)$ at $t = \frac{1}{2}$. (c) Where does this tangent line intersect the z-axis ?* [10]

$$\mathbf{r}(t) = \begin{bmatrix} t \\ 2t \\ 1+t^2 \end{bmatrix}.$$

$x = t$, $y = 2t$ $\Rightarrow$ $y = 2x$

$x = t$, $z = 1+t^2$

(a)




(b) $z = 1+x^2$

$$\underline{r}(t) = \begin{bmatrix} t \\ 2t \\ 1+t^2 \end{bmatrix}, \quad \underline{r}'(t) = \begin{bmatrix} 1 \\ 2 \\ 2t \end{bmatrix}$$

at $t = \frac{1}{2}$: $\underline{r}'(\frac{1}{2}) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\underline{r}(\frac{1}{2}) = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 1+\frac{1}{4} \end{bmatrix}$

$$\underline{r} = \begin{bmatrix} \frac{1}{2} \\ 1 \\ 5/4 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \begin{aligned} x &= \tfrac{1}{2} + t \\ y &= 1 + 2t \\ z &= 5/4 + t \end{aligned}$$ elim $t$: $\dfrac{x-\frac{1}{2}}{1} = \dfrac{y-1}{2} = \dfrac{z-\frac{5}{4}}{1}$

(c) on z-axis: $x = 0$, $y = 0$

$0 - \frac{1}{2} = \frac{0-1}{2} = \frac{z-5/4}{1}$

raaklyn: *tangent line:*

$$\boxed{\frac{x-\frac{1}{2}}{1} = \frac{y-1}{2} = \frac{z-\frac{5}{4}}{1}}$$

$z - \frac{5}{4} = -\frac{1}{2}$

$z = -\frac{1}{2} + \frac{5}{4} = \frac{3}{4}$

sny z-as by: *intersects z-axis at:* $\boxed{3/4}$

B3 Beskou die oppervlak S en die vlak P. *Consider the surface S and the plane P.* [8]

$$\text{S: } z = y^2e^x, \qquad \text{P: } 4x + 2y - \frac{1}{4}z = 7.$$

Vind een punt op S waar die raakvlak aan S parallel aan P is. Vind dan ook die vergelyking van daardie raakvlak.

*Find one point on S where the tangent plane to S is parallel to P. Also find the equation of that tangent plane.*

$\underline{n}$ for the plane P is $\begin{bmatrix} 4 \\ 2 \\ -\frac{1}{4} \end{bmatrix}$ ✓

S: $G = y^2e^x - z$, when $G = 0$. Some scaling ✓

$\nabla G = \begin{bmatrix} y^2e^x \\ 2ye^x \\ -1 \end{bmatrix}$ ✓ Point is when $\nabla G = \alpha \underline{n}$

$$\begin{bmatrix} y^2e^x \\ 2ye^x \\ -1 \end{bmatrix} = \begin{bmatrix} 4\alpha \\ 2\alpha \\ -\frac{1}{4}\alpha \end{bmatrix}$$

i.e. $-1 = -\frac{1}{4}\alpha$, $\alpha = 4$ ✓

$y^2e^x = 16$

$2ye^x = 8$

$4^2e^x = 16.$

$\rightarrow x = 0$ ✓

$\frac{y^2e^x}{2ye^x} = \frac{16}{8}$, $\frac{y}{2} = 2$

$y = 4$ ✓

point is $\begin{bmatrix} 0 \\ 4 \\ 4^2e^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 16 \end{bmatrix} = \underline{r}_1$ ✓

Tangent plane at $\underline{r}_1$ is:

$$\begin{bmatrix} 4 \\ 2 \\ -\frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -\frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 4 \\ 16 \end{bmatrix}$$

$4x + 2y - \frac{1}{4}z = 0 + 8 - 4$ ✓

$\boxed{4x + 2y - \frac{1}{4}z = 4}$

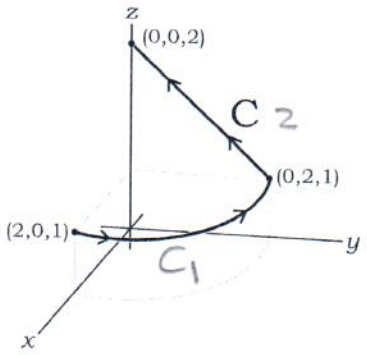
or $16x + 8y - z = 16$

B4 Bereken die integraal $U$ hieronder. Let op dat $\mathbf{F}$ nie konserwatief is nie. C bestaan uit 'n kwart sirkelboog met radius 2 in 'n horisontale vlak, gevolg deur 'n reguitlynstuk. Begin- en eindpunt-koördinate word gegee sowel as die koördinate van die aansluitpunt tussen die twee stukke van C.

*Calculate the integral $U$ below. Note that $\mathbf{F}$ is not conservative. C consists of a quarter arc of a circle in a horizontal plane, followed by a straight line segment. The coordinates of the starting point, the end point as well as the point where the two sections of C meet, are given.*

$$\mathbf{F} = -3xy\,\mathbf{i} + 2z\,\mathbf{k}, \qquad U = \int_C \mathbf{F}\cdot d\mathbf{r}$$

10




$$U = \int_C \underline{F}\cdot d\underline{r} = \int_C -3xy\,dx + 2z\,dz$$

$$= \int_0^{\pi/2} -3(2\cos t)(2\sin t)(-2\sin t\,dt) + 2\cdot 1(0) + \int_0^1 0 + 2(1+t)\,dt$$

$C_1$: $x = 2\cos t$, $y = 2\sin t$, $z = 1$; $t \in [0, \frac{\pi}{2}]$

$C_2$: $x = 0 + 0t$, $y = 2 + (-2)t$, $z = 1 + t$; $t \in [0, 1]$

$$= 24\int_0^{\pi/2} \sin^2 t\cos t\,dt + \int_0^1 (2+2t)\,dt$$

$$= 24\left.\frac{(\sin t)^3}{3}\right|_0^{\pi/2} + \left[2t + \frac{2t^2}{2}\right]_0^1$$

$$= 8 - 0 + 2 + 1 - 0 = 11$$

EINDE VAN VRAESTEL

*END OF PAPER*