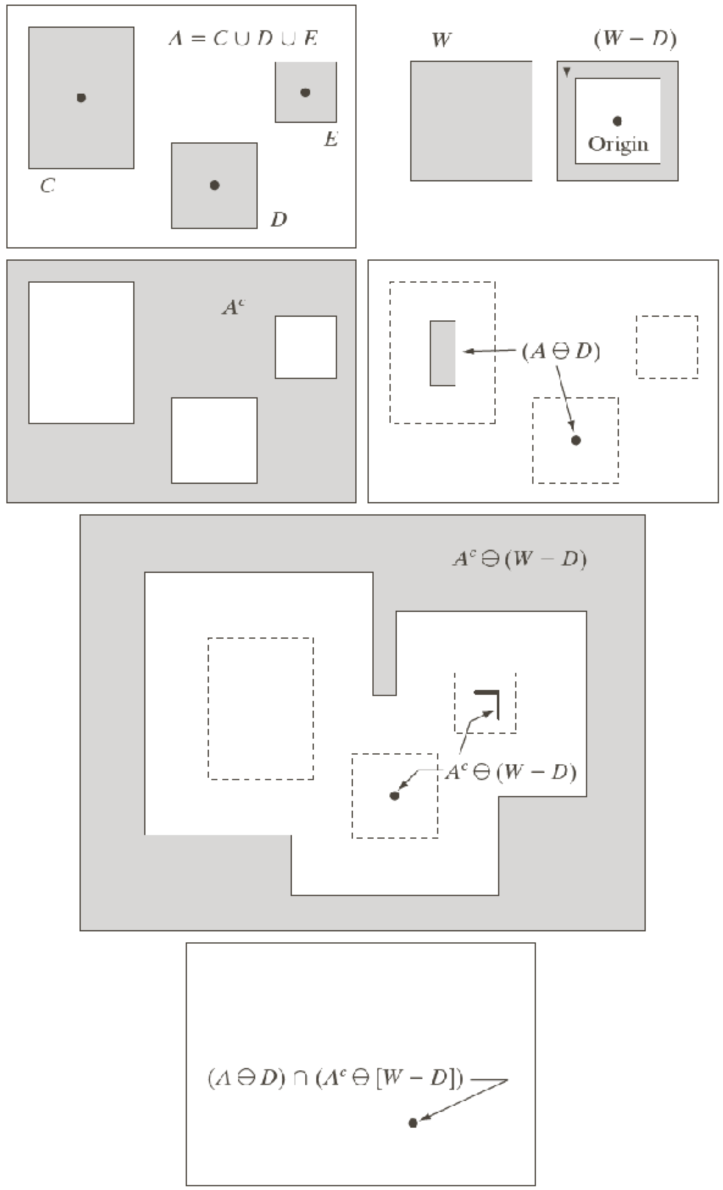
9.4 The hit-or-miss transformation **Illustration...**



a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

- **Objective is to find a disjoint region (set) in an image**
- **If B denotes the set composed of D and its background, the match/hit (or set of matches/hits) of B in A, is**

$$A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

- **Generalized notation:** $B = (B_1, B_2)$
  - B_1: **Set formed from elements of B associated with an object**
  - B_2: **Set formed from elements of B associated with the corresponding background**

  [Preceeding discussion: $B_1 = D$ **and** $B_2 = (W - D)$**]**

  - **More general definition:**

$$A \circledast B = (A \ominus B_1) \cap [A^c \ominus B_2]$$

  - $A \circledast B$ **contains all the origin points at which, simultaneously,** B_1 **found a hit in** A **and** B_2 **found a hit in** A^c

- **Alternative definition:**

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

- **A background is necessary to detect disjoint sets**

- **When we only aim to detect certain patterns within a set, a background is not required, and simple erosion is sufficient**

9.5 Some basic morphological algorithms

When dealing with binary images, the principle application of morphology is extracting image components that are useful in the representation and description of shape

9.5.1 Boundary extraction

The boundary $\beta(A)$ of a set A is

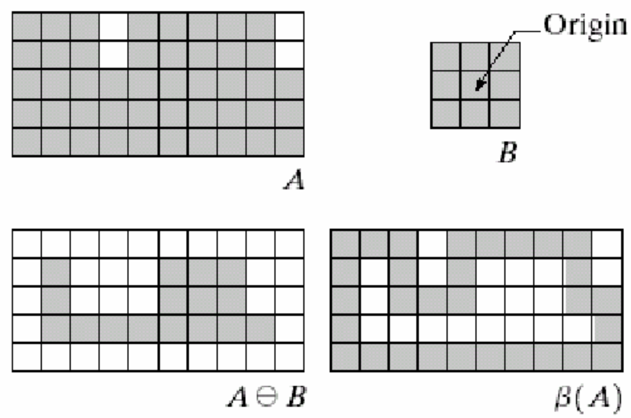
$$\beta(A) = A - (A \ominus B),$$

where B is a suitable structuring element

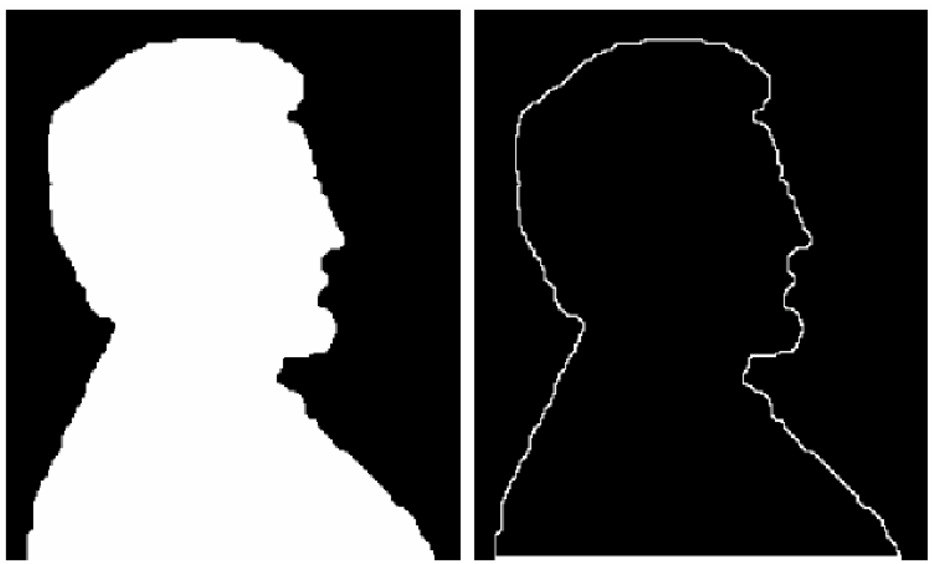
Illustration...

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Example 9.5: Morphological boundary extraction



a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

9.5.2 Hole filling

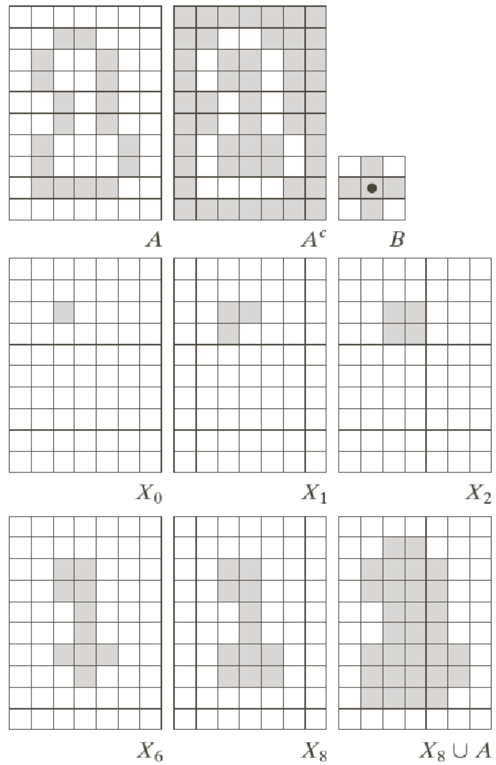
- $A \equiv$ set whose elements are 8-connected boundaries that enclose a background region (hole)
- Given a point p in each hole, the objective is to fill all the holes with 1's
- All non-boundary (background) points are labeled 0
- Begin by forming an array X_0 of 0's, except at the locations in X_0 that correspond to the points p in each hole, which is set to 1...
- The following procedure fills all the holes with 1's,

$$X_k = (X_{k-1} \oplus B) \cap A^c, \quad k = 1, 2, 3, \ldots,$$

where B is the symmetric structuring element in figure 9.15 (c)

- The algorithm terminates at iteration step k if $X_k = X_{k-1}$
- The set union of X_k and A contains the filled set and its boundary

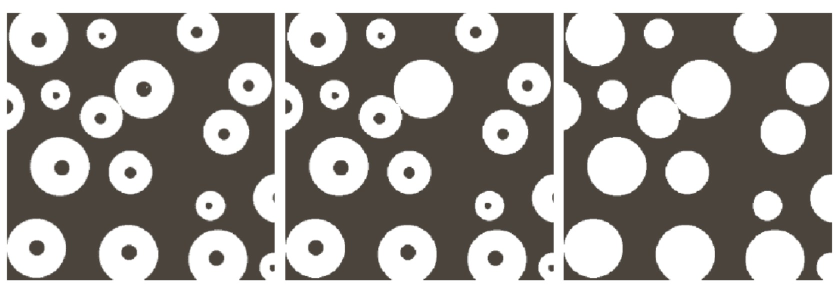
Note that the intersection at each step with A^c limits the dilation result to inside the region of interest



a b c
d e f
g h i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Example 9.6: Morphological hole filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

9.5.3 Extraction of connected components

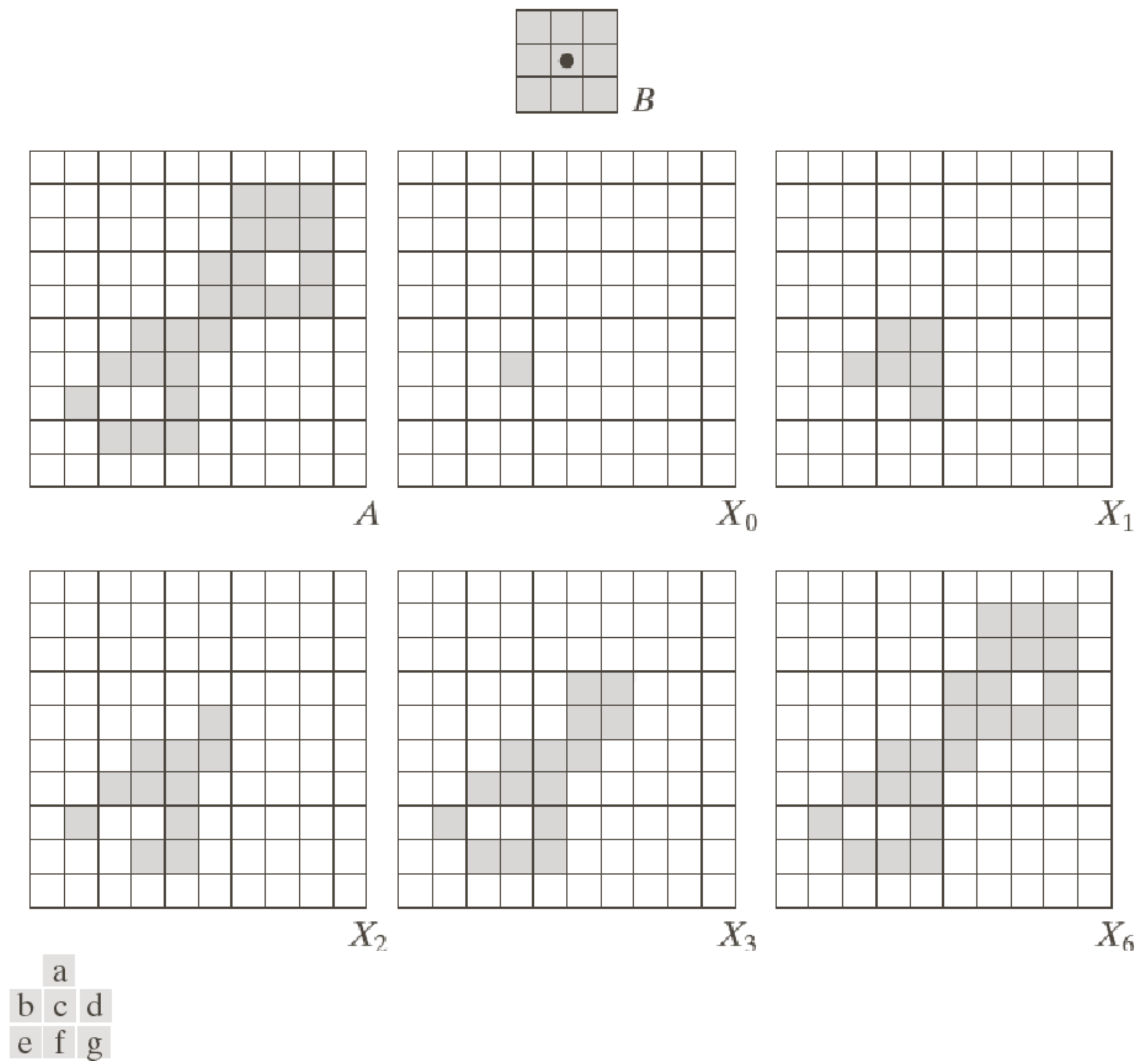
Let A be a set containing one or more connected components, and form an array X_0 (with the same size as A) whose elements are 0 (background), except at each location known to correspond to a point in each connected component in A, which is set to 1 (foreground)

The following iterative procedure starts with X_0 and find all the connected components

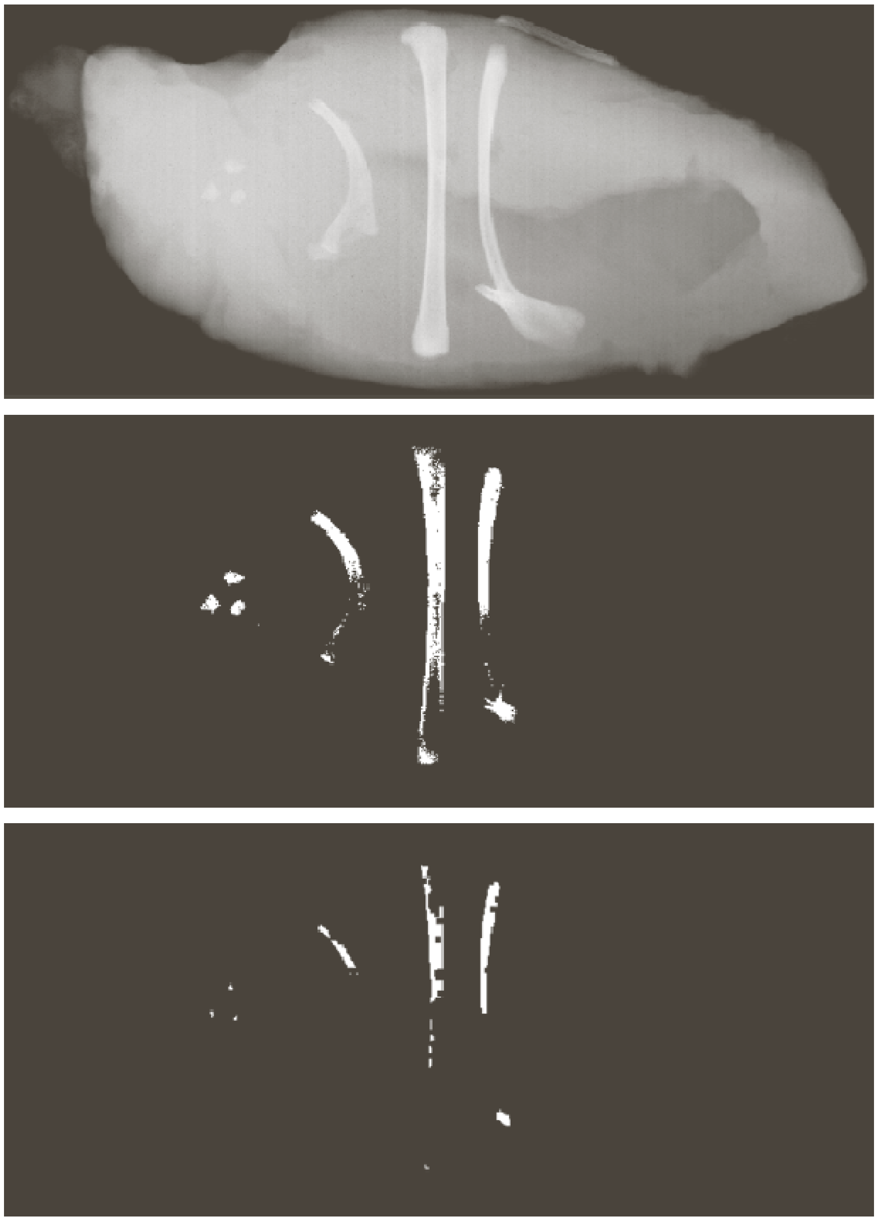
$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots,$$

where B is a suitable structuring element. When $X_k = X_{k-1}$, with X_k containing all the connected components, the procedure terminates

This algorithm is applicable to any finite number of sets of connected components contained in A, assuming that a point is known in each connected component


B
A
X_0
X_1
X_2
X_3
X_6
a
b c d
e f g

Example 9.7



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

9.5.4 Convex hull

Morphological algorithm for obtaining the convex hull, $C(A)$, of a set A...

Let B^1, B^2, B^3 and B^4 represent the four structuring elements in Fig 9.19 (a), and then implement the equation ...

$$X_k^i = (X_{k-1} \circledast B^i) \cup A,\ i = 1, 2, 3, 4,\ k = 1, 2, \ldots,\ X_0^i = A$$

Now let $D^i = X_{\text{conv}}^i$, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. Then the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

Procedure illustrated in Fig 9.19: $\times$ entries indicate "don't care" conditions

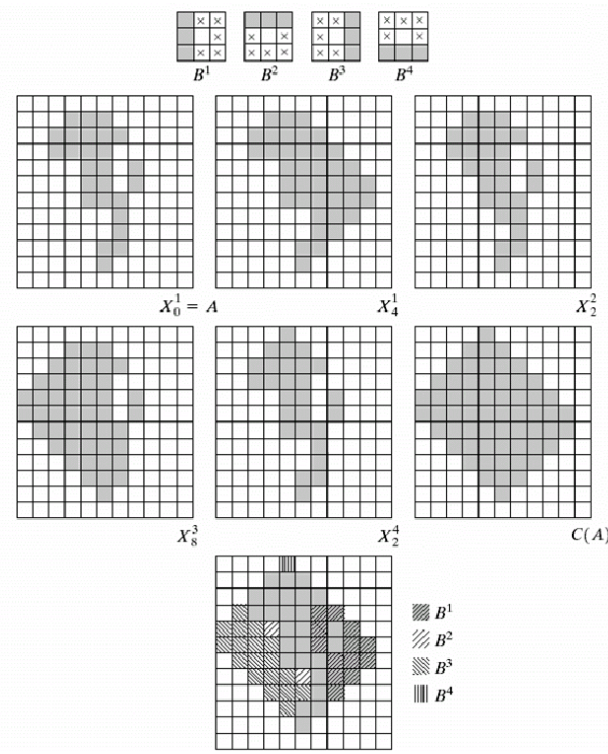
Shortcoming of above algorithm: convex hull can grow beyond the minimum dimensions required to guarantee convexity

Possible solution: Limit growth so that it does not extend past the vertical and horizontal dimensions of the original set of points

Boundaries of greater complexity can be used to limit growth even further in images with more detail

a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



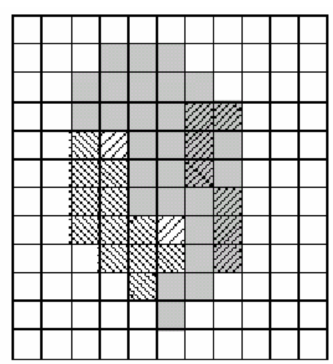


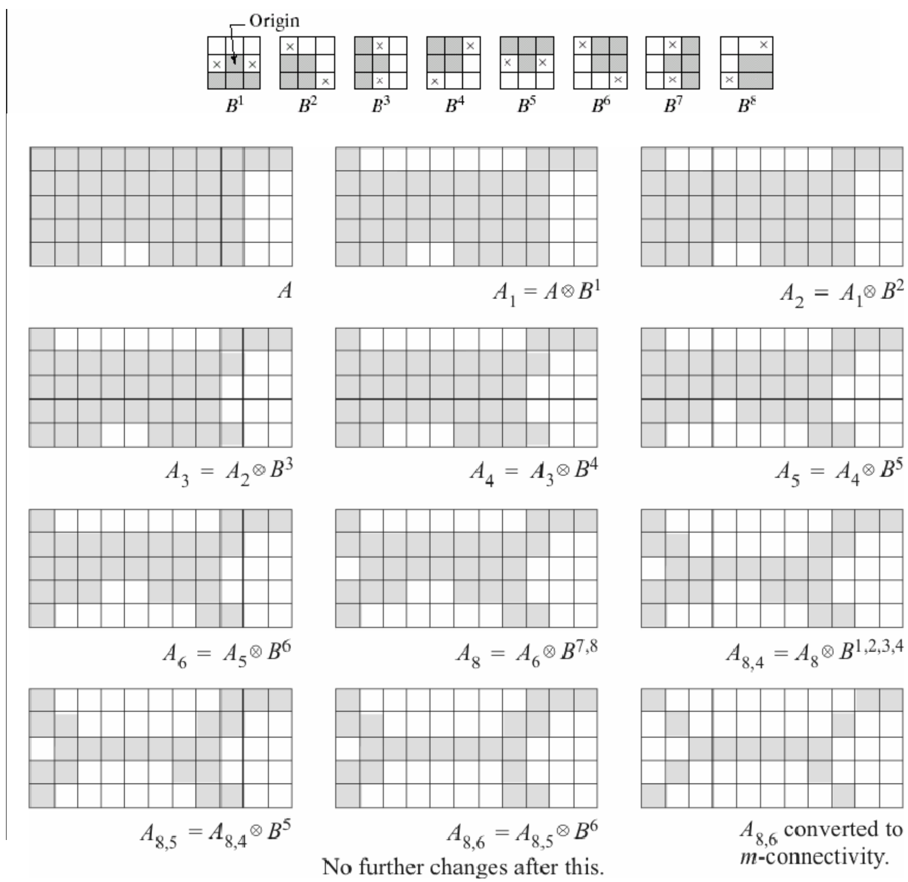
FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

9.5.5 Thinning: The thinning of a set A by a structuring element B:

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$

Symmetric thinning: Sequence of SEs, $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$, where B^i is a rotated version of B^{i-1}

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$$



9.5.6 Thickening: **Thickening is the morphological dual of thinning and is defined by:**

$$A \odot B = A \cup (A \circledast B),$$

where B is a structuring element

Similar to thinning: $A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2) \ldots) \odot B^n)$

Structuring elements for thickening are similar to those of Fig 9.21 (a), but with all 1's and 0's interchanged

A separate algorithm for thickening is seldom used in practice – we thin the background instead, and then complement the result

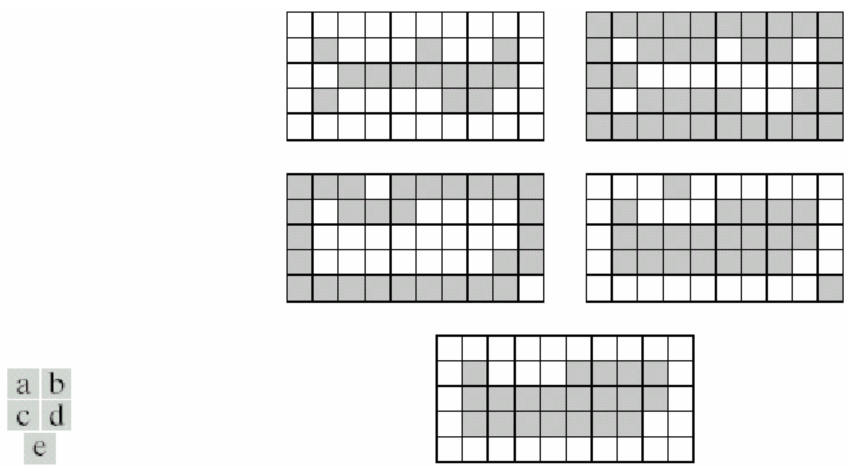
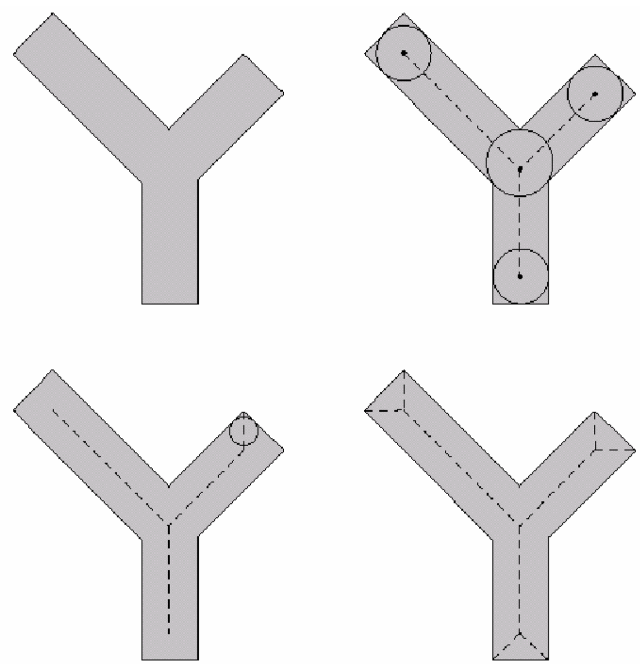


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

9.5.7 Skeletons

The algorithm proposed in this section is similar to the medial axis transformation (MAT). The MAT transformation is discussed in section 11.1.7 and is far inferior to the skeletonization algorithm introduced in section 11.1.7. The skeletonization algorithm proposed in this section also does not guarantee connectivity. We therefore do not discuss this algorithm.

Illustration of the above comments...



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

A further illustration...

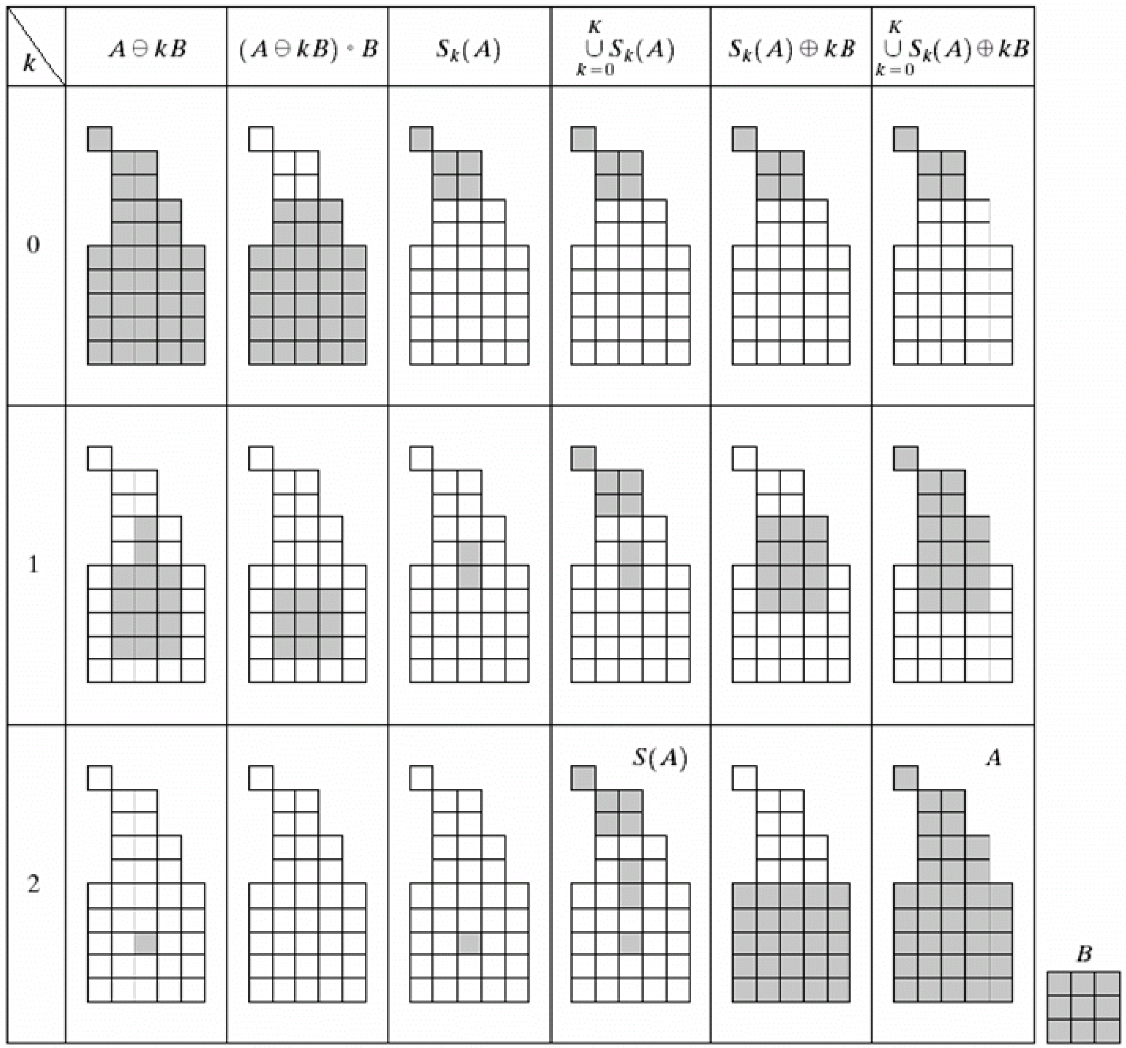


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

9.5.8 Pruning

- Cleans up "parasitic" components left by thinning and skeletonization
- Use combination of morphological techniques

Illustrative problem: Hand-printed character recognition

- Analyze shape of skeleton of character
- Skeletons characterized by spurs ("parasitic" components)
- Spurs caused during erosion of non-uniformities in strokes
- We assume that the length of a parasitic component does not exceed a specified number of pixels

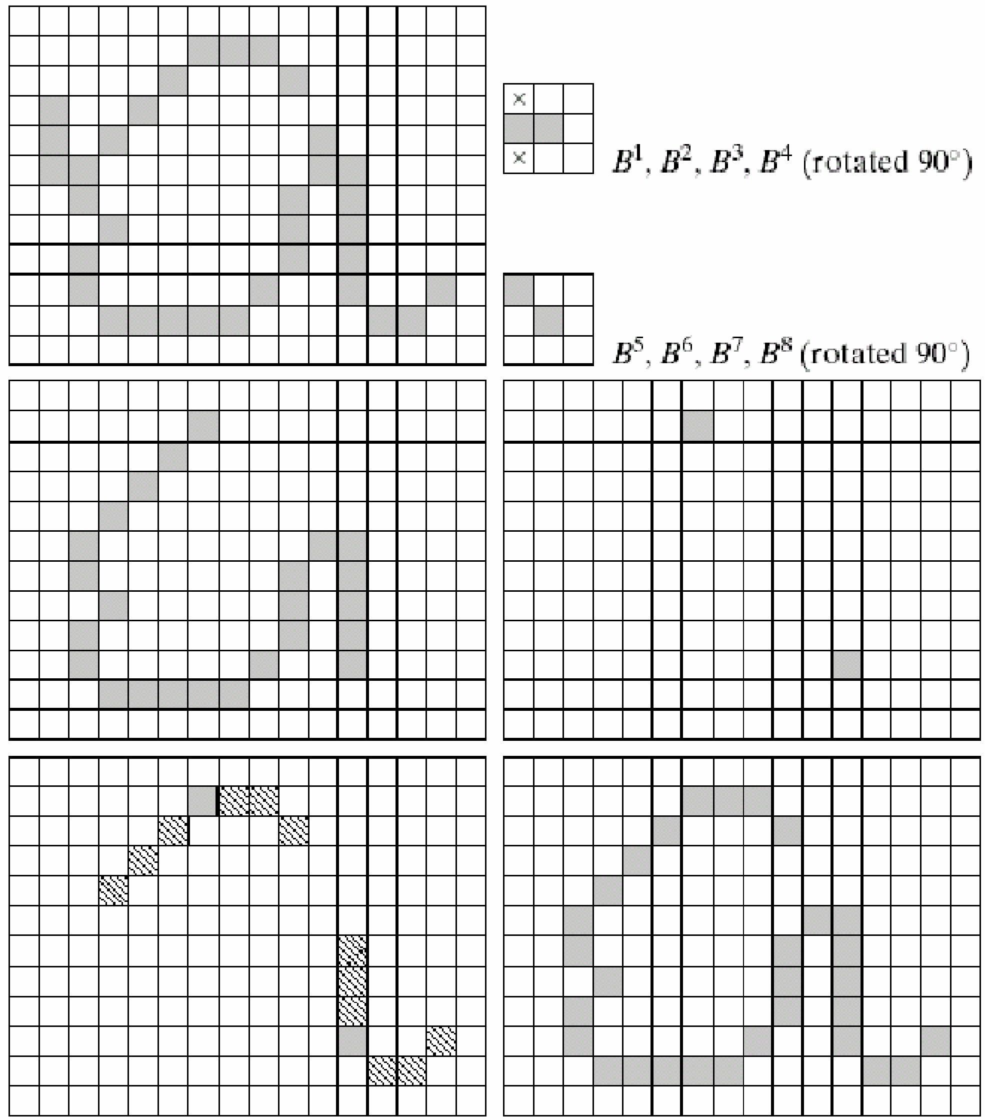


FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Any branch with three or less pixels is to be eliminated

(1) Three iterations of:

$$X_1 = A \otimes \{B\}$$

(2) Find all the end points in X_1:

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

(3) Dilate end points three times, using A as a delimiter:

$$X_3 = (X_2 \oplus H) \cap A, \quad H = \begin{array}{|c|c|c|} \hline \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \hline \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \hline \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \hline \end{array}$$

(4) Finally:

$$X_4 = X_1 \cup X_3$$