NOTES ON SOME USEFUL LOGICAL TERMS AND RULES

A statement P is a declarative sentence that is true or false but not both. If P is a true statement, then its truth value is true, and if it is a false statement, then its truth value if false.

write 1 for true and 0 for false

The negation ~P of P has truth table

P	~P
0	1
1	0

~P reads "not P"

The disjunction P ∨ Q of statements P and Q has truth table

P	Q	P ∨ Q
0	0	0
0	1	1
1	0	1
1	1	1

P∨Q reads "P or Q"

The conjunction P ∧ Q of statements P and Q has truth table

P	Q	P ∧ Q
0	0	0
0	1	0
1	0	0
1	1	1

P∧Q reads "P and Q

Two statements are <u>logically equivalent</u> if they have the same truth table. For example, De Morgan's laws state that

$\sim(P \vee Q)$ is logically equivalent to $(\sim P) \wedge (\sim Q)$

and $\sim(P \wedge Q)$ is logically equivalent to $(\sim P) \vee (\sim Q)$.

We may check these laws with truth tables:

P	Q	$P \vee Q$	$\sim(P \vee Q)$	$\sim P$	$\sim Q$	$(\sim P) \wedge (\sim Q)$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Since $\sim(P \vee Q)$ and $(\sim P) \wedge (\sim Q)$ have the same truth table, they are logically equivalent.

Exercise: Check the other De Morgan law with truth tables.

Implication $P \Rightarrow Q$ has truth table

P	Q	$P \Rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

$P \Rightarrow Q$ reads "P implies Q" or "If P, then Q".

Exercise: Show with truth tables that $P \Rightarrow Q$ and $(\sim P) \vee Q$ are logically equivalent.

An <u>open sentence</u> $P(n)$ is a declarative sentence, the truth value of which depends on the variable n.

Example: $P(n) =$ "$2n$ is divisible by 6" has truth values $P(0) = 1$, $P(1) = 0$, $P(2) = 0$, $P(3) = 1$, etc.

An open sentence $P(n)$ may be converted into a <u>quantified statement</u> in two ways:

1. $\forall n, P(n)$ reads "For every n, $P(n)$" and $\forall$ is called the <u>universal quantifier</u>.

2. $\exists n, P(n)$ reads "There exists an n such that $P(n)$" and $\exists$ is called the <u>existential quantifier</u>.

Example: If $P(n)$ is the open sentence "$2n$ is divisible by 6", then

$\forall n \in \mathbb{Z}, P(n)$ is a false statement while
$\exists n \in \mathbb{Z}, P(n)$ is a true statement.