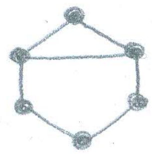
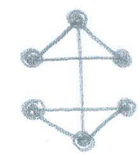
Assignment 3

#1 (3.2)

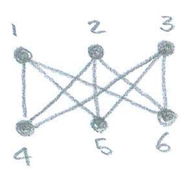
G: 

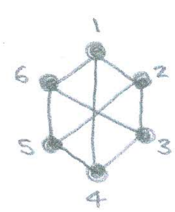
H: 

K: 

All three of these graphs have order 6, size 7 and degree sequence 3, 3, 2, 2, 2, 2.
$H \ncong G$ and $H \ncong K$, since G and K each contains three mutually adjacent vertices (a triangle), whereas H does not. Also, G contains C_6 as subgraph, while K does not; therefore $G \ncong K$.

#2 (3.4)

G_1: 

G_2: 

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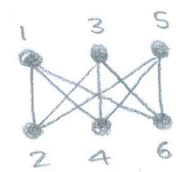
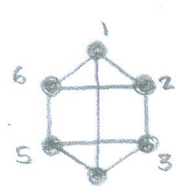
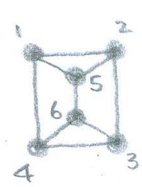


$G_1 \cong G_2$; an isomorphism ϕ from G_1 to G_2 is given by $\phi(1)=1$, $\phi(2)=3$, $\phi(3)=5$, $\phi(4)=2$, $\phi(5)=4$, $\phi(6)=6$.

G_3: 

G_4: 

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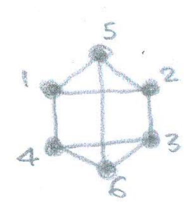


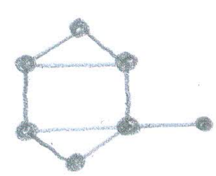
$G_3 \cong G_4$; an isomorphism ψ from G_3 to G_4 is given by $\psi(1)=5$, $\psi(2)=2$; $\psi(3)=3$, $\psi(4)=6$, $\psi(5)=4$, $\psi(6)=1$.

G_1 is bipartite, while G_3 is not bipartite since it contains triangles, which are odd cycles. Therefore G_1 (and G_2) is not isomorphic to G_3 (and G_4).

#3 (3.6)

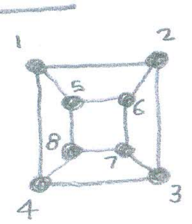
No. The graph G contains a vertex of degree 2 that is adjacent to a vertex of degree 3 and a vertex of degree 4 as well as a vertex of degree 2 that is adjacent to two vertices of degree 3.

G :

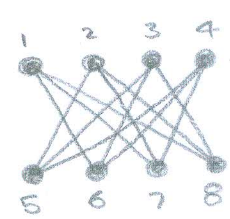


#4 (3.8)

G_1 :



G_2 :



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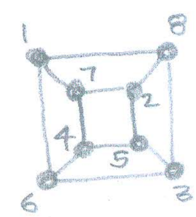

$G_1 \cong G_2$; an isomorphism ϕ from G_1 to G_2 is given by $\phi(1)=1$, $\phi(2)=8$, $\phi(3)=3$, $\phi(4)=6$, $\phi(5)=7$, $\phi(6)=2$, $\phi(7)=5$, $\phi(8)=4$.

G_2 is bipartite, while G_3 has odd cycles. Therefore $G_2 \ncong G_3$, and therefore $G_1 \ncong G_3$ also.

#5 (3.10)

No. If G is disconnected, then $\bar{G}$ is connected (by Theorem 1.11), and therefore $G \ncong \bar{G}$.

#6 (3.12)

Note that, since H is self-complementary of even order n, $\deg_H(v) < \frac{n}{2}$ if and only if $\deg_{\bar{H}}(v) \geq \frac{n}{2}$.

Since G and H are self-complementary graphs, there exist isomorphisms $\phi_1 : V(G) \to V(\bar{G})$ and $\phi_2 : V(H) \to V(\bar{H})$. Define $\phi : V(F) \to V(\bar{F})$ by

$$\phi(u) = \begin{cases} \phi_1(u) & \text{if } u \in V(G) \\ \phi_2(u) & \text{if } u \in V(H). \end{cases}$$

We show that ϕ is an isomorphism from F to $\bar{F}$, by showing that uv is an edge of F if and only if $\phi(u)\phi(v)$ is an edge of $\bar{F}$:

<u>case 1</u> u and v both in $V(G)$: Since ϕ_1 is an isomorphism, $uv \in E(G)$ if and only if $\phi_1(u)\phi_1(v) \in E(\bar{G})$. Therefore, by the definition of ϕ, $uv \in E(F)$ if and only if $\phi(u)\phi(v) \in E(\bar{F})$.

<u>case 2</u> u and v both in $V(H)$: similar argument as that of case 1.

<u>case 3</u> u in G and v in H: $uv \in E(F)$ if and only if $\deg_H(v) < \frac{n}{2}$ if and only if $\deg_{\bar{H}}(v) \geq \frac{n}{2}$ if and only if $\phi(u)\phi(v) = \phi_1(u)\phi_2(v) \in E(\bar{F})$.