
Problem 1: Classify each of the following linear PDEs as hyperbolic, parabolic or elliptic:

(a) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial x \partial y} + 3\dfrac{\partial^2 u}{\partial y^2} = 0$

(b) $\dfrac{\partial^2 u}{\partial x^2} = 9\dfrac{\partial^2 u}{\partial x \partial y}$

(c) $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial u}{\partial x} - 6\dfrac{\partial u}{\partial y} = 0$

***Probleem 1:** Klassifiseer elkeen van die volgende lineêre PDVs as hiperbolies, parabolies of ellipties:*

(a) $\dfrac{\partial^2 u}{\partial x^2} - \dfrac{\partial^2 u}{\partial x \partial y} + 3\dfrac{\partial^2 u}{\partial y^2} = 0$

(b) $\dfrac{\partial^2 u}{\partial x^2} = 9\dfrac{\partial^2 u}{\partial x \partial y}$

(c) $\dfrac{\partial^2 u}{\partial x^2} + 2\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial u}{\partial x} - 6\dfrac{\partial u}{\partial y} = 0$

Problem 2: Solve the heat equation ($u_t = ku_{xx}$), subject to the following conditions:

***Probleem 2:** Los die hitte-vergelyking ($u_t = ku_{xx}$) op, onderhewig aan die volgende voorwaardes:*

$$u(0,t) = 0, \quad u(L,t) = 0, \quad u(x,0) = \begin{cases} 1, & 0 < x < \frac{L}{2} \\ 0, & \frac{L}{2} < x < L \end{cases}$$

Hint: use the result derived in Lecture 11.

Wenk: gebruik die resultaat wat in Lesing 11 herlei is.

Problem 3: Find an expression for the temperature $u(x,t)$ in a rod of length L, if the initial temperature is $f(x)$ through the rod and if the ends $x = 0$ and $x = L$ are both **thermally insulated** (see bottom of p. 463).

Hint: Tutorial 2, Problem 5 may prove useful here.

Also investigate the behaviour of $u(x,t)$ as $t \to \infty$, with reference to the physical situation.

***Probleem 3:** Vind 'n uitdrukking vir die temperatuur $u(x,t)$ in 'n staaf van lengte L, as die aanvanklike temperatuur deur die staaf $f(x)$ is, en die endpunte $x = 0$ en $x = L$ albei **termies geïsoleer** is (sien bl. 463, onderaan).*

Wenk: Tutoriaal 2, Probleem 5 kan hier nuttig wees.

Ondersoek ook die gedrag van $u(x,t)$ soos $t \to \infty$, met verwysing na die fisiese situasie.

Problem 4: Use separation of variables to find product solutions for the following PDEs:

(a) $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y}$

(b) $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = u$

(c) $x\dfrac{\partial u}{\partial x} = y\dfrac{\partial u}{\partial y}$

***Probleem 4:** Gebruik skeiding van veranderlikes om produkoplossings vir die volgende PDVs te vind:*

(a) $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y}$

(b) $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = u$

(c) $x\dfrac{\partial u}{\partial x} = y\dfrac{\partial u}{\partial y}$
